

Modélisation d'un drone convertible et analyse de platitude - premiers résultats

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What is a convertible aircraft ?

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A combination between:

- a multicopter



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- a fixed-wing aircraft



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- hover at fixed point
- take-off and landing vertically
 - ↳ reduced environmental footprint (no need for a runway)

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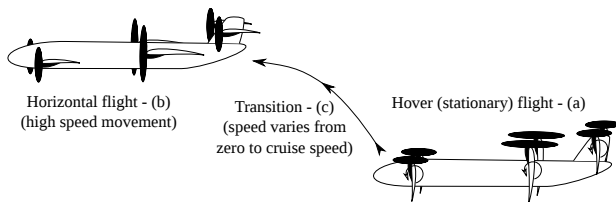
- hover at fixed point
- take-off and landing vertically
 - ↳ reduced environmental footprint (no need for a runway)

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- improve aerodynamic efficiency using wings
 - ↳ reduced energy consumption
 - ↳ increased flight length

Proposed convertible aircraft concept

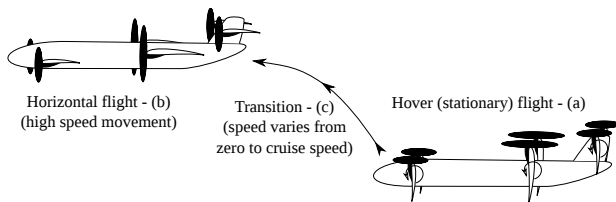


- 6 propeller-engines: 4 for VTOL and 2 for forward flight
- 3 pairs of wings: canard, main and elevator
- tilt-wing design: wings and propellers are fixed together

Project goal: design autonomous pilot for safe transition

1. modelling of the dynamic behaviour
 - initial work on the modelling (gimbal lock problem)
2. robust and fault tolerant control
 - flatness analysis of the model

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Part I – Dynamic model

Dynamic model – reference frames

- inertial frame (\mathcal{I})
- vehicle-carried normal Earth frame (\mathcal{O})
- body frame (\mathcal{B})
- aerodynamic (\mathcal{A}) and kinematic (\mathcal{K}) frames are supposed identical (zero wind speed)

Euler angles are used to represent vector rotations between frames:

- from \mathcal{B} to \mathcal{O} :

$$\begin{aligned} R_b^o &= R_z(\psi(t))R_y(\theta(t))R_x(\phi(t)) \\ &= \begin{bmatrix} c\psi c\theta & -s\psi c\phi + c\psi s\theta s\phi & s\psi s\phi + c\psi s\theta c\phi \\ s\psi c\theta & c\psi c\phi + s\psi s\theta s\phi & -c\psi s\phi + s\psi s\theta c\phi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix} \end{aligned}$$

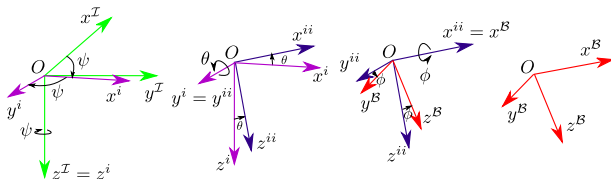
- from \mathcal{A} to \mathcal{O} : $R_a^o = R_z(\chi(t))R_y(\gamma(t))R_x(\mu(t))$
- from \mathcal{A} to \mathcal{B} : $R_a^b = R_y(-\alpha(t))R_z(\beta(t))$

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Dynamic model – Newton's second law

Newton's second law in a non-inertial frame \mathcal{A} :

$$\dot{\xi} = R_a^o V_a^a, \quad (1)$$

$$m \frac{dV_a^a}{dt} + \Omega_{ao}^a \wedge m V_a^a = F^a + (R_a^o)^T G^o. \quad (2)$$

- $\xi(t) = [x(t) \ y(t) \ z(t)]^T$: position of the aircraft in frame \mathcal{I}
- $V_a^a(t) = [v_a(t) \ 0 \ 0]^T$: speed vector in frame \mathcal{A}
- $\Omega_{ao}^a(t)$: rotational speed of frame \mathcal{A} with respect to frame \mathcal{O}
- m : mass of the aircraft

$$F^a(t) = \begin{bmatrix} X^a(t) \\ Y^a(t) \\ Z^a(t) \end{bmatrix} = F_a^a(t) + \left(R_a^b\right)^T F_p^b(t) \quad (3)$$

- $F_a^a(t)$: aerodynamic forces
- $F_p^b(t)$: propulsion forces

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Dynamic model – Euler's second law

Euler's rotation equations:

$$\frac{d(I\Omega_{bo}^b(t))}{dt} + \Omega_{bo}^b(t) \wedge I\Omega_{bo}^b(t) = \tau_b(t). \quad (4)$$

- I : inertia matrix
- $\Omega_{bo}^b(t) = [p(t) \ q(t) \ r(t)]^T$: rotational speed of frame \mathcal{B} wrt frame \mathcal{O}

$$\tau^b(t) = \begin{bmatrix} L^b(t) \\ M^b(t) \\ N^b(t) \end{bmatrix} = \tau_a^b(t) + M_p^b(t) + \tau_d^b(t) \quad (5)$$

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- $M_p^b(t)$: moments from propulsion forces
- $\tau_d^b(t)$: drag couples

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Dynamic model – Euler angles derivatives

Time evolution of Euler angles between frames \mathcal{A} and \mathcal{O} is obtained using:

$$\dot{R}_a^o = R_a^o [\Omega_{ao}^a]_{\times} \quad (6)$$

Skew-symmetric operator $[\cdot]_{\times}$, of a vector $W = [w_1 \ w_2 \ w_3]^T \in \mathbb{R}^3$:

$$[W]_{\times} = \begin{bmatrix} 0 & -w_3 & w_2 \\ w_3 & 0 & -w_1 \\ -w_2 & w_1 & 0 \end{bmatrix}. \quad (7)$$

Using that

$$\Omega_{ao} = \Omega_{ab} + \Omega_{bo} \implies \Omega_{ao}^a = \Omega_{ab}^a + R_b^a \Omega_{bo}^b, \quad (8)$$

while

$$\dot{R}_a^b = R_a^b [\Omega_{ab}^a]_{\times} \implies [\Omega_{ab}^a]_{\times} = (R_a^b)^T \dot{R}_a^b, \quad (9)$$

which implies

$$\Omega_{ab}^a = [-\dot{\alpha}s\beta \quad -\dot{\alpha}c\beta \quad \dot{\beta}]^T. \quad (10)$$

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Dynamic model – system of equations

Newton's 2nd law:

$$\begin{cases} \dot{x}(t) = c\chi c\gamma v_a, \\ \dot{y}(t) = s\chi c\gamma v_a, \\ \dot{z}(t) = -s\gamma v_a, \\ \dot{v}_a(t) = \frac{X^a}{m} - s\gamma g, \\ \dot{\beta}(t) = s\alpha p - c\alpha r + \frac{c\gamma s\mu mg + Y^a}{mv_a}, \\ \dot{\alpha}(t) = q - (c\alpha p + s\alpha r)t\beta + \frac{c\gamma c\mu}{c\beta} \frac{g}{v_a} + \frac{Z^a}{c\beta mv_a} \end{cases}$$

Euler angles derivatives:

$$\begin{cases} \dot{\chi}(t) = \frac{-Z^a s\mu + Y^a c\mu}{v_a mc\gamma}, \\ \dot{\gamma}(t) = \frac{-c\gamma gm - Y^a s\mu - Z^a c\mu}{v_a m}, \\ \dot{\mu}(t) = \frac{-c\mu c\gamma s\beta g}{v_a c\beta} + \frac{p c\alpha + r s\alpha}{c\beta} - \frac{Z^a s\beta}{v_a mc\beta} + \frac{s\gamma(Y^a c\mu - Z^a s\mu)}{v_a mc\gamma}, \end{cases}$$

Euler's 2nd law:

$$\begin{cases} \dot{p}(t) = \frac{(I_{xz}(I_{xx} - I_{yy} + I_{zz})p - (I_{xz}^2 - I_{zz}(I_{yy} - I_{zz}))r)q}{I_{xx}I_{zz} - I_{xz}^2} + \frac{I_{xz}N^b + I_{zz}L^b}{I_{xx}I_{zz} - I_{xz}^2}, \\ \dot{q}(t) = \frac{-I_{xz}p^2 - r(I_{xx} - I_{zz})p + I_{xz}r^2 + M^b}{I_{yy}}, \\ \dot{r}(t) = \frac{((I_{xz}^2 + I_{xx}(I_{xx} - I_{yy}))p - I_{xz}(I_{xx} - I_{yy} + I_{zz})r)q}{I_{xx}I_{zz} - I_{xz}^2} + \frac{I_{xx}N^b + I_{xz}L^b}{I_{xx}I_{zz} - I_{xz}^2}. \end{cases}$$

How to handle the gimbal lock?

Euler angles: gimbal lock at $\gamma(t) = 90^\circ$

➡ problem during vertical take-off and landing

Solution: use a second set of Euler angles $\mu_v(t)$, $\gamma_v(t)$, and $\chi_v(t)$ known as vertical Euler angles (Castillo et al. (2005))

$$R_{av}^o = R_y(\pi/2)R_z(\chi_v(t))R_y(\gamma_v(t))R_x(\mu_v(t)) \quad (11)$$

Part II – Flatness analysis of the model

Recall on flat systems (J. Lévine, 2009):

$$\dot{x} = f(x, u), \quad x \in \mathbb{R}^n, \quad u \in \mathbb{R}^m, \quad m \leq n$$

is differentially flat iff $\exists y \in \mathbb{R}^m, y = f_y(x, u, \dot{u}, \dots, u^{(p)})$, s.t.

$$x = f_x(y, \dot{y}, \ddot{y}, \dots, y^{(q)}),$$

$$u = f_u(y, \dot{y}, \ddot{y}, \dots, y^{(q+1)}).$$

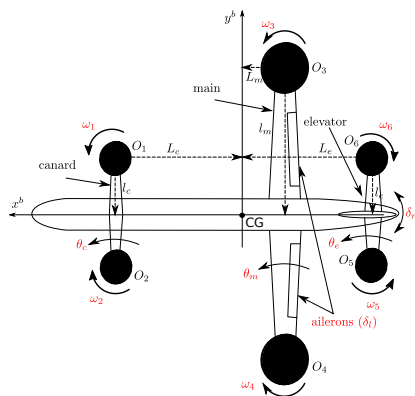
→ y are called the flat outputs.

Applications of flatness: feedforward control, fault detection and isolation (FDI), ...

Flatness analysis – control inputs

The convertible aircraft has a total of 11 control inputs:

- 3 wing rotation angles (θ_c , θ_m , and θ_e),
- rotation speeds of 6 propeller-engines (ω_1 , ω_2 , ω_3 , ω_4 , ω_5 , and ω_6),
- main wing aileron deflection (δ_l),
- rudder tilt angle (δ_n)



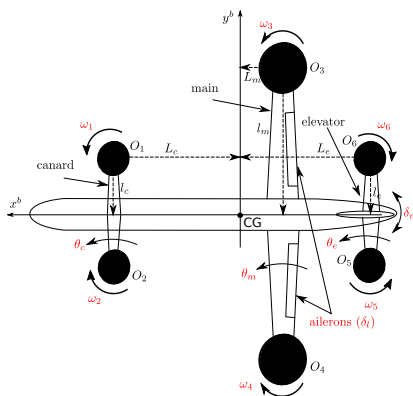
There are only 5 independent control inputs:

- Take-off, landing, and hover
↳ ω_1 , ω_2 , ω_5 , ω_6 , and $\theta_c = \theta_e$
- Fast forward flight
↳ ω_3 , ω_4 , δ_l , δ_n , and θ_c

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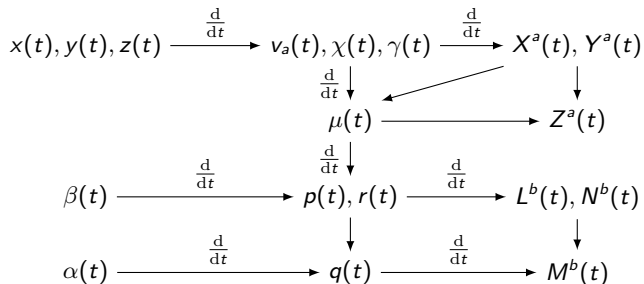


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Flatness analysis – flat outputs

Proposed set of flat outputs: $x(t)$, $y(t)$, $z(t)$, $\alpha(t)$, and $\beta(t)$.



What happens if $v_a(t)$ is small?

Problem: model written using vertical Euler angles becomes singular.

Proposed solution: write Newton's second law in frame \mathcal{B}

$$\dot{\xi} = V_o^o, \quad (12)$$

$$m \frac{dV_o^o}{dt} = R_b^o F_p^b + G^o, \quad (13)$$

$$\dot{R}_b^o = R_b^o \left[\Omega_{bo}^b \right]_{\times}, \quad (14)$$

$$\frac{d(I\Omega_{bo}^b(t))}{dt} = -\Omega_{bo}^b(t) \wedge I\Omega_{bo}^b(t) + \tau_b(t). \quad (15)$$

where $F_p^b(t) = R_y(\theta_e(t)) [F_p^e(t) \ 0 \ 0]^T$.

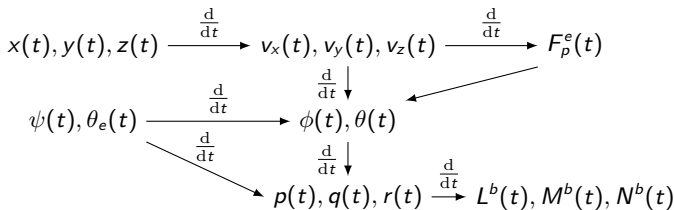
$$\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{z}(t) \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} \quad (16a)$$

$$\begin{bmatrix} \dot{v}_x(t) \\ \dot{v}_y(t) \\ \dot{v}_z(t) \end{bmatrix} = R_z(\psi)R_y(\theta)R_x(\phi)R_y(\theta_e) \begin{bmatrix} F_p^e \\ 0 \\ 0 \end{bmatrix} + G^o \quad (16b)$$

$$\begin{bmatrix} \dot{\phi}(t) \\ \dot{\theta}(t) \\ \dot{\psi}(t) \end{bmatrix} = \begin{bmatrix} 1 & s\phi t\theta & c\phi t\theta \\ 0 & c\phi & -s\phi \\ 0 & \frac{s\phi}{c\theta} & \frac{c\phi}{c\theta} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \quad (16c)$$

$$I \begin{bmatrix} \dot{p}(t) \\ \dot{q}(t) \\ \dot{r}(t) \end{bmatrix} = - \begin{bmatrix} p \\ q \\ r \end{bmatrix} \wedge I \begin{bmatrix} p \\ q \\ r \end{bmatrix} + \begin{bmatrix} L^b \\ M^b \\ N^b \end{bmatrix} \quad (16d)$$

Flat outputs: $x(t)$, $y(t)$, $z(t)$, $\psi(t)$, and $\theta_e(t) = \theta_c(t)$.



Conclusions and perspectives

- In this paper: convertible aircraft modelling with focus on the flatness property
- Other models can be obtained (e.g., using quaternion rotation)
 - ▶ need to analyse flatness
 - ▶ might be useful for simulation
- Further work is necessary to determine the sizes the convertible (3D modelling and CFD)
- Fault tolerant guidance of the convertible aircraft to be adressed next

Thank you for your attention!

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