Direct adaptive rejection of unknown time-varying narrow band disturbances applied to a benchmark problem

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***Abstract*—The paper presents a direct adaptive algorithm for the rejection of unknown time-varying narrow band disturbances, applied to an adaptive regulation benchmark. The objective is to minimize the residual force by applying an appropriate control signal on the inertial actuator in the presence of multiple and/or unknown time-varying disturbances. The direct adaptive control algorithm is based on the internal model principle (IMP) and uses the Youla-Kučera (YK) parametrization. A direct feedback adaptive regulation is proposed and evaluated both in simulation and real-time. The robustness is improved by shaping the sensitivity functions of the system through band stop filters (BSF).**

***Index Terms*—Adaptive Regulation, Active Vibration Control, Inertial Actuators, Multiple Narrow Band Disturbances, Youla-Kučera Parametrization, Internal Model Principle**

I. Introduction

A basic problem in active vibration control is the attenuation (rejection) of multiple narrow band disturbances of unknown and time-varying frequencies. The energy of these disturbances (or vibrations) is concentrated in a narrow band around an unknown frequency and could be modelled as a white noise or a Dirac impulse passed through a *model of the disturbance*. While, in general, one can assume a certain structure for such *model of disturbance*, its parameters are unknown and may be time-varying. The need of an adaptive approach arises.

This problem could be addressed using additional transducers to obtain an *image* of the perturbation (inspired by Widrow's technique for adaptive noise cancellation [29]) considering that the measurement is highly correlated with the unknown disturbance. This leads to a *feedforward* approach as in [5], [10], [11], [14]. The disadvantages of this approach are:

- Requires the use of an additional transducer.
- Difficult choice for the location of this transducer.
- Requires the adaptation of many parameters.

A *feedback* approach overcome these disadvantages providing disturbance rejection (at least asymptotically), using only the measurement of the residual force (acceleration) as in [1], [2], [22].

Different problem scenarios have been considered using a feedback solution and the *model of the disturbance*:

1) Unknown plant and disturbance model [12].
2) Unknown plant and known disturbance model [26], [30].
3) Known plant and unknown disturbance model [7], [1], [2], [28], [24], [9], [15], [16]

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The present paper belongs to the third category, since the dynamic characteristics of the system are practically constant for a given physical realization. Also, very reliable estimates of the parameters of the *control model* can be obtained by standard system identification.

In this context, the following approaches have been considered for solving the problem of feedback regulation of unknown and time-varying narrow-band disturbances:

1) Use of the internal model principle [1], [2], [4], [13], [15], [16], [17], [22], [27], [28].
2) Use of an observer for the disturbance [9], [24], [8].
3) Use of the *phase-locked* loop structure considered in communications systems [7], [6].

Since the model of the perturbation is consider unknown, an adaptive implementation is required. Two approaches can be considered: *direct* or *indirect*.

Direct adaptive schemes require less computational time than indirect schemes and through the Youla-Kučera (YK) parametrization of the controller along with the Internal Model Principle (IMP), offer advantages as simpleness, excellent adaptation transients and closed loop stability during the adaptive regulation. This approach has been successfully used in a number of applications [22], [21], and therefore has been considered to be applied to the benchmark.

The YK parametrization (known also as the Q-parametrization) allows to insert and adjust the internal model of the disturbance into the controller by adjusting the parameters of the polynomial $\hat{Q}(z^{-1})$ (see Fig. 3). The above is done without recomputing the *central controller* ($R_0(z^{-1})$ and $S_0(z^{-1})$ in Fig. 3 remain unchanged). This also preserves the *desired* closed loop poles adding to the robustness of the control scheme. Also the number of parameters to be directly adapted is roughly equal to the number of parameters on the denominator disturbance model. The above means that the size of the adaptation algorithm will depend upon the complexity of the disturbance model.

The design that will be presented in this paper preserves the fixed parts in the nominal controller (as shown in [21]) and also shows how the robustness of the closed loop can be improved by chosing adequate Band-Stop Filters (BSF) for shaping the sensitivity functions.

The paper is organized as follows. Section II presents briefly the active suspension system using an inertial actuator on which the algorithms will be tested. Section III presents the general plant and controller structure in the context of the YK parametrization. The direct adaptive algorithm is revised in Section IV. Section V discusses briefly the design of the central controller using BSFs for shaping the sensitivity functions. Simulation results are presented in Section VI, while experimental results for this methodology are given in Section VII. Concluding remarks are presented in Section VIII.

II. Active Vibration Control System using an Inertial Actuator

The structure of the system used for the benchmark on adaptive regulation is presented in Fig. 1. A picture of the real system is presented in Fig. 2, on both figures the structure components are indicated. The system consist of a passive damper (indicated with the number 2 in Fig. 1), an inertial actuator (indicated with the number 3), a load, a transducer for the residual force, a controller, a power amplifier and a shaker (source of perturbation and indicated with the number 1). The control objective is to reject the effect of unknown narrow-band disturbances on the output of the system (residual force), *i.e.*, to attenuate the vibrations transmitted from the machine to the chassis. The transfer function $(N_p(z^{-1})/D_p(z^{-1}))$, between the disturbance force $\delta(t)$ and the residual force $y(t)$ is called *primary path*. The plant transfer function $(z^{-d}B(z^{-1})/A(z^{-1}))$ between the input of the inertial actuator , $u(t)$, and the residual force is called *secondary path*. The system is controlled in real-time using a MATLAB xPC target environment. The sampling frequency is $F_s = 800$ Hz. For a detailed hardware description see [20], [19]. In [19] the LTI discrete time models for both paths are given[1]. The secondary path model presents a number of low damped vibration modes as well as low damped complex zeros (anti-resonance).

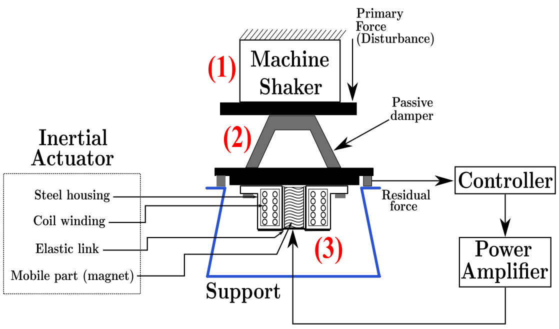

Fig. 1. Active vibration control (scheme)

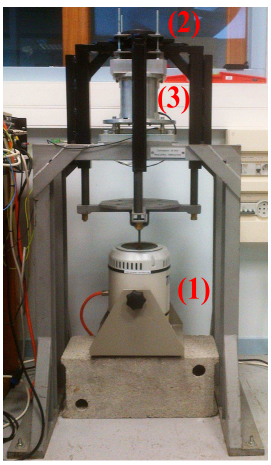

Fig. 2. Active vibration control (photograph)

[1]The primary path model is given for simulation purposes.

III. Plant Representation and Controller Structure

The structure of the LTI discrete time model of the plant, also called *secondary path*, used for controller design is

$$G(z^{-1}) = \frac{z^{-d}B(z^{-1})}{A(z^{-1})} = \frac{z^{-d-1}B^*(z^{-1})}{A(z^{-1})}, \quad (1)$$

where

$$A(z^{-1}) = 1 + a_1 z^{-1} + \cdots + a_{n_A} z^{-n_A}, \quad (2)$$
$$B(z^{-1}) = b_1 z^{-1} + \cdots + b_{n_B} z^{-n_B} = z^{-1} B^*, \quad (3)$$
$$B^* = b_1 + \cdots + b_{n_B} z^{-n_B+1}, \quad (4)$$

and d is the plant pure time delay in number of sampling periods[2].

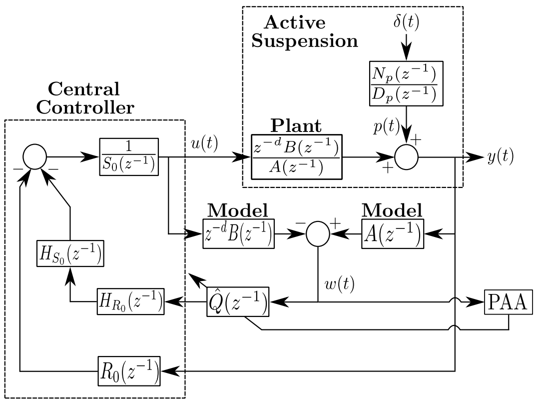

Fig. 3. Direct adaptive regulation scheme for rejection of unknown disturbances

Without considering a reference signal, the output of the plant $y(t)$ and the input $u(t)$ may be written as (see Fig. 3):

$$y(t) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})} \cdot u(t) + p(t), \quad (5)$$
$$S_0(q^{-1}) \cdot u(t) = -R_0(q^{-1}) \cdot y(t). \quad (6)$$

In (5), $p(t)$ is the effect of the disturbances on the measured output[3] and $R_0(z^{-1})$, $S_0(z^{-1})$ are polynomials in z^{-1} having the following expressions[4]:

$$S_0 = 1 + s_1^0 z^{-1} + \ldots + s_{n_S}^0 z^{-n_S} = S_0'(z^{-1}) \cdot H_{S_0}(z^{-1}), \quad (7)$$
$$R_0 = r_0^0 + r_1^0 z^{-1} + \ldots + r_{n_R}^0 z^{-n_R} = R_0'(z^{-1}) \cdot H_{R_0}(z^{-1}), \quad (8)$$

where $H_{S_0}(q^{-1})$ and $H_{R_0}(q^{-1})$ represent pre-specified parts of the controller (used for example to incorporate the internal model of a disturbance or to open the loop at certain frequencies) and $S_0'(q^{-1})$ and $R_0'(q^{-1})$ are computed.

[2]The complex variable z^{-1} will be used to characterize the system's behaviour in the frequency domain and the delay operator q^{-1} will be used for the time domain analysis.

[3]The disturbance passes through a so called *primary path* which is represented in this figure, and $p(t)$ is its output.

[4]The argument (z^{-1}) will be omitted in some of the following equations to make them more compact.

We define the output sensitivity function (the transfer function between the disturbance $p(t)$ and the output of the system $y(t)$) as

$$S_{yp}(z^{-1}) = \frac{A(z^{-1})S_0(z^{-1})}{P_0(z^{-1})} \tag{9}$$

and the input sensitivity function (the transfer function between the disturbance $p(t)$ and the control input $u(t)$) as

$$S_{up}(z^{-1}) = -\frac{A(z^{-1})R(z^{-1})}{P_0(z^{-1})}, \tag{10}$$

where

$$P_0(z^{-1}) = A(z^{-1})S_0(z^{-1}) + z^{-d}B(z^{-1})R_0(z^{-1}), \tag{11}$$

the characteristic polynomial, specifies the desired closed loop poles of the system[5] (see also [23]). It is important to remark that one should only reject disturbances located in frequency regions where the plant model has enough gain. This can be seen by looking at eq. (9) and noticing that perfect rejection at a certain frequency, ω_0, is obtained *iff* $S_0(e^{-j\omega_0}) = 0$. On the other hand, from eq. (10) one can see that this has a bad effect on the control input if the gain of the secondary path is too small at ω_0.

In this paper, the Youla-Kučera parametrization ([3], [27]) is used. Supposing a finite impulse response (FIR) representation of the adaptive Q filter

$$Q(z^{-1}) = q_o + q_1 z^{-1} + \ldots + q_{n_Q} z^{-n_Q} \tag{12}$$

the controller's polynomials are:

$$R = R_0 + AQH_{S_0}H_{R_0}, \tag{13}$$
$$S = S_0 - z^{-d}BQH_{S_0}H_{R_0}. \tag{14}$$

where R_0 and S_0 define the central controller which verifies the desired specifications in the absence of the disturbance. The characteristic polynomial of the closed loop becomes

$$P = AS_0 + z^{-d}BR_0. \tag{15}$$

IV. Direct Adaptive Regulation for Disturbance Rejection

This section presents the direct adaptive control algorithm ([22], [21]) that will be used for the benchmark problem. A key aspect of this methodology is the use of the IMP. It is supposed that $p(t)$ is a deterministic disturbance given by

$$p(t) = \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t), \tag{16}$$

where $\delta(t)$ is a Dirac impulse and N_p, D_p are coprime polynomials of degrees n_{N_p} and n_{D_p}, respectively[6]. In the case of stationary narrow-band disturbances, the roots of $D_p(z^{-1})$ are on the unit circle and the contribution of the terms of N_p can be neglected.

Internal Model Principle: The effect of the disturbance given in (16) upon the output

$$y(t) = \frac{A(q^{-1})S(q^{-1})}{P(q^{-1})} \cdot \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t), \tag{17}$$

where $D_p(z^{-1})$ is a polynomial with roots on the unit circle and $P(z^{-1})$ is an asymptotically stable polynomial, converges asymptotically towards zero *iff* the polynomial $S(z^{-1})$ in the *RS* controller has the form (based on eq. (7))

$$S(z^{-1}) = D_p(z^{-1})H_{S_0}(z^{-1})S'(z^{-1}). \tag{18}$$

Thus, the pre-specified part of $S(z^{-1})$ should be chosen as $H_S(z^{-1}) = D_p(z^{-1})H_{S_0}(z^{-1})$ and the controller is computed solving

$$P = AD_pH_{S_0}S' + z^{-d}BH_{R_0}R', \tag{19}$$

where P, D_p, A, B, H_{R_0}, H_{S_0} and d are given[7].

To compute $Q(z^{-1})$ in order that the polynomial $S(z^{-1})$ given by (14) incorporates the internal model of the disturbance (18), one has to solve the diophantine equation

$$S'D_p + z^{-d}BH_{R_0}Q = S'_0, \tag{20}$$

where D_p, d, B, S'_0, and H_{R_0} are known and S' and Q are unknown. Eq. (20) has a unique solution for S' and Q with: $n_{S'_0} \leq n_{D_p} + n_B + d + n_{H_{R_0}} - 1$, $n_{S'} = n_B + d + n_{H_{R_0}} - 1$, $n_Q = n_{D_p} - 1$. One sees that the order n_Q of the polynomial Q depends upon the structure of the disturbance model. The use of the Youla-Kučera parametrization, with Q given in (12), is interesting because it allows to maintain the closed loop poles as given by the central controller but at the same time introduce the parameters of the internal model into the controller. The development of the parametric adaptation algorithm (PAA) requires first find an *error equation* (see also [27], [22], [21]). Using the Q-parametrization, the output of the system in the presence of a disturbance can be expressed as

$$\begin{aligned} y(t) &= \frac{A[S_0 - q^{-d}BH_{S_0}H_{R_0}Q]}{P} \cdot \frac{N_p}{D_p} \cdot \delta(t) \\ &= \frac{S_0 - q^{-d}BH_{S_0}H_{R_0}Q}{P} \cdot w(t), \end{aligned} \tag{21}$$

where $w(t)$ is given by (see also Fig. 3)

$$w(t) = \frac{AN_p}{D_p} \cdot \delta(t) = A \cdot y(t) - q^{-d} \cdot B \cdot u(t). \tag{22}$$

Taking into consideration that the adaptation of Q is done in order to obtain an output $y(t)$ which tends asymptotically to zero, one can define $\varepsilon^0(t+1)$ as the value of $y(t+1)$ obtained with $\hat{Q}(t,q^{-1})$ (the estimate of Q at time t, written also $\hat{Q}(t)$)

$$\varepsilon^0(t+1) = \frac{S_0}{P} \cdot w(t+1) - \hat{Q}(t)\frac{q^{-d}B^*H_{S_0}H_{R_0}}{P} \cdot w(t). \tag{23}$$

Similarly, the *a posteriori* error becomes (using $\hat{Q}(t+1)$) as

$$\varepsilon(t+1) = \frac{S_0}{P} \cdot w(t+1) - \hat{Q}(t+1)\frac{q^{-d}B^*H_{S_0}H_{R_0}}{P} \cdot w(t). \tag{24}$$

[5]It is assumed that a reliable model identification is achieved and therefore the estimated model is assumed to be equal to the true model.

[6]Throughout the paper, n_X denotes the degree of the polynomial X.

[7]Of course, it is assumed that D_p and B do not have common factors.

Replacing S_0 from the last equation by (20), one obtains

$$\varepsilon(t+1) = [Q - \hat{Q}(t+1)] \cdot \frac{q^{-d} B^* H_{S_0} H_{R_0}}{P} \cdot w(t) + v(t+1), \quad (25)$$

where

$$v(t) = \frac{S' D_p H_{S_0}}{P} \cdot w(t) = \frac{S' H_{S_0} A N_p}{P} \cdot \delta(t) \quad (26)$$

is a signal which tends asymptotically towards zero.

Define the estimated polynomial $\hat{Q}(t,q^{-1}) = \hat{q}_0(t) + \hat{q}_1(t)q^{-1} + \ldots + \hat{q}_{n_Q}(t)q^{-n_Q}$ and the associated estimated parameter vector $\hat{\theta}(t) = [\hat{q}_0(t)\ \hat{q}_1(t)\ \ldots\ \hat{q}_{n_Q}(t)]^T$. Define the fixed parameter vector corresponding to the optimal value of the polynomial Q as: $\theta = [q_0\ q_1\ \ldots\ q_{n_Q}]^T$.

Denote

$$w_2(t) = \frac{q^{-d} B^* H_{S_0} H_{R_0}}{P} \cdot w(t) \quad (27)$$

and define the following observation vector

$$\phi^T(t) = [w_2(t)\ w_2(t-1)\ \ldots\ w_2(t-n_Q)]. \quad (28)$$

Eq. (25) becomes

$$\varepsilon(t+1) = [\theta^T - \hat{\theta}^T(t+1)] \cdot \phi(t) + v(t+1). \quad (29)$$

One can remark that $\varepsilon(t+1)$ corresponds to an adaptation error ([18]).

From eq. (23), one obtains the *a priori* adaptation error

$$\varepsilon^0(t+1) = w_1(t+1) - \hat{\theta}^T(t)\phi(t), \quad (30)$$

with

$$w_1(t+1) = \frac{S_0(q^{-1})}{P(q^{-1})} \cdot w(t+1), \quad (31)$$

$$w(t+1) = A(q^{-1}) \cdot y(t+1) - q^{-d} B^*(q^{-1}) \cdot u(t), \quad (32)$$

where $B(q^{-1})u(t+1) = B^*(q^{-1})u(t)$.

The *a posteriori* adaptation error is obtained from (24)

$$\varepsilon(t+1) = w_1(t+1) - \hat{\theta}^T(t+1)\phi(t). \quad (33)$$

For the estimation of the parameters of $\hat{Q}(t,q^{-1})$ the following PAA is used ([18]):

$$\hat{\theta}(t+1) = \hat{\theta}(t) + F(t)\phi(t)\varepsilon(t+1), \quad (34)$$

$$\varepsilon(t+1) = \frac{\varepsilon^0(t+1)}{1 + \phi^T(t)F(t)\phi(t)}, \quad (35)$$

$$\varepsilon^0(t+1) = w_1(t+1) - \hat{\theta}^T(t)\phi(t), \quad (36)$$

$$F(t+1) = \frac{1}{\lambda_1(t)} \left[F(t) - \frac{F(t)\phi(t)\phi^T(t)F(t)}{\frac{\lambda_1(t)}{\lambda_2(t)} + \phi^T(t)F(t)\phi(t)} \right], \quad (37)$$

$$1 \geq \lambda_1(t) > 0,\ 0 \leq \lambda_2(t) < 2, \quad (38)$$

where $\lambda_1(t)$, $\lambda_2(t)$ allow to obtain various profiles for the evolution of the adaptation gain $F(t)$ (for details see [18], [23]).

V. Central Controller Design

The central controller plays a relevant role in this approach, making of it a key element. Its role is to stabilize the system in the absence of disturbances and to allow to obtain a small amplification outside the attenuation frequencies, when the adaptive regulation algorithm is active. The central controller was presented in eqs. (7) and (8) and is indicated in Fig. 3.

Since the amplifications outside the attenuation frequencies are reflected in the output sensitivity function ($S_{yp}(z^{-1})$), the technique of pole placement with sensitivity function shaping is an option to address the problem (see details in [23]). The benchmark specifications propose a frequency region of interest from 50 to 95 Hz. Considering this region as a *band* it is possible to shape the sensitivity functions through a *band-pass filter* approach, in order to minimize the effect of the IMP design. This is achieved by introducing fixed auxiliary low damped poles near to the frequency limits of the band of interest. The damping factor of these poles has to be chosen in a way that the effect of the IMP will not be eliminated, just attenuated. To preserve the robustness and dynamics, all the poles of the system are conserved.

From eqs. (8) and (7), the central controller can incorporate fixed parts for specific purposes. In [21] it is shown how to preserve the fixed parts of the central controller using a YK parametrization. Following this approach and considering the characteristics of BSFs (see details in [23] and [25]), a reduction in the modulus of $S_{up}(z^{-1})$ for higher frequencies can be imposed (for example, see Fig. 4) without introducing undesired effects on $S_{yp}(z^{-1})$. This improves the robustness by reducing the effects of the IMP design outside the frequency band of interest.

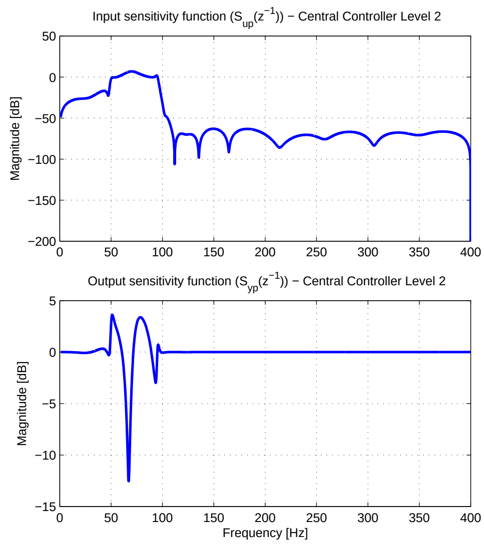

Fig. 4. Sensitivity functions for Level 2

To accomplish this, the BSFs' transfer function numerator ($n_{BSF}(z^{-1})$) is implemented on the fixed part of the controller's numerator polynomial[8],

$$H_{R_0} = H'_{R_0} n_{BSF},$$

while the BSF transfer function denominator ($d_{BSF}(z^{-1})$) will define additional closed loop poles. No fixed parts were considered for $S_0(z^{-1})$.

VI. Simulation Results

According to [20], [19], the benchmark's specifications consider three levels in terms of the number of narrow band disturbances to be rejected (attenuated). For each level, three types of tests were designed for which performance specifications have to be achieved.

The first series of tests, called *Simple Step Test*, deals with global attenuation (GA in dB), disturbance attenuation (DA in dB), maximum amplification (MA in dB) outside the attenuation frequencies (these quantities are evaluated once the adaptation has settled), transient duration (TD in sec), maximum value (MV in Volts) during transient, a measure of the integral of the square error during transient (N^2T), and after settling of the adaptation (N^2R). The second series of tests consider the evaluation of performances for step changes in frequencies and the third consider the evaluation of performances when the disturbances are chirp signals.

A. Level 1 Results

The Table I shows the results obtained in the presence of one sinusoidal disturbance with constant frequency (Simple Step Test). The benchmark specifications for global attenuation and disturbance attenuation[9] are passed at all frequencies. The maximum amplifications obtained are slightly over the limit (6 dB), while the transient duration is lower than 2 seconds. The Table II resume the results obtained when the disturbance frequency changes (Step Changes Frequency Test) showing in general fast transients (lower than 2 seconds) without increasing the maximum values.

The third test consist in applying a chirp signal as disturbance. The results are in the Table III. The two periods of chirp are indicated as $\nearrow$ (for increasing frequency) and $\searrow$ (for decreasing frequency). The benchmark requirement is that the maximum value of the residual force (output of the system) is not greater than 0.1 V. One can see that the benchmark specifications are satisfied.

B. Level 2 Results

The Level 2 increases the difficulty but the compromise achieved between attenuation and amplification is demonstrated in Table I, where the simulation results for simple step test are presented. Both specifications, global attenuation and disturbance attenuation, were satisfied. The maximum amplification is in some cases marginally over the limit (7 dB for this level). For step changes frequency test, the increments in the transient time (see Table IV) do not pass the benchmark limit. The chirp requirement is also fulfilled for this level (Table V).

[8] H'_{R_0} is used for opening the loop at $0f_s$ and $0.5f_s$

[9] At 95 Hz the disturbance attenuation is marginally below the benchmark specification for the three levels.

TABLE II
Simulation Results - Level 1 - Step Changes Frequency Test

SEQUENCE - 1			
Frequency (Hz)	**Norm^2 Trans.** ($\times 10^{-3}$)	**Max. Val.** ($\times 10^{-3}$)	**Transient (msec)**
$60 \rightarrow 70$	17.127	22.155	110.00
$70 \rightarrow 60$	16.948	16.101	106.25
$60 \rightarrow 50$	46.205	18.719	143.75
$50 \rightarrow 60$	41.759	30.724	143.75
SEQUENCE - 2			
Frequency (Hz)	**Norm^2 Trans.** ($\times 10^{-3}$)	**Max. Val.** ($\times 10^{-3}$)	**Transient (msec)**
$75 \rightarrow 85$	16.738	15.657	132.50
$85 \rightarrow 75$	15.422	18.571	126.25
$75 \rightarrow 65$	14.178	15.091	121.25
$65 \rightarrow 75$	14.246	17.137	103.75
SEQUENCE - 3			
Frequency (Hz)	**Norm^2 Trans.** ($\times 10^{-3}$)	**Max. Val.** ($\times 10^{-3}$)	**Transient (msec)**
$85 \rightarrow 95$	39.322	23.835	30.00
$95 \rightarrow 85$	32.884	28.047	98.75
$85 \rightarrow 75$	15.447	21.127	93.75
$75 \rightarrow 85$	16.307	15.769	126.25

TABLE III
Simulation Results - Level 1 - Chirp Test

	Error-Mean Square Value	**Error-Maximum Value**
$\nearrow$	14.499×10^{-6}	13.963×10^{-3}
$\searrow$	14.0×10^{-6}	14.860×10^{-3}

TABLE IV
Simulation Results - Level 2 - Step Changes Frequency Test

SEQUENCE - 1			
Frequency (Hz)	**Norm^2 Trans.** ($\times 10^{-3}$)	**Max. Val.** ($\times 10^{-3}$)	**Transient (msec)**
$[55,75] \rightarrow [60,80]$	38.334	34.663	208.75
$[60,80] \rightarrow [55,75]$	36.202	30.359	206.25
$[55,75] \rightarrow [50,70]$	78.016	35.463	263.75
$[50,70] \rightarrow [55,75]$	70.110	39.072	165.00
SEQUENCE - 2			
Frequency (Hz)	**Norm^2 Trans.** ($\times 10^{-3}$)	**Max. Val.** ($\times 10^{-3}$)	**Transient (msec)**
$[70,90] \rightarrow [75,95]$	60.982	30.850	262.50
$[75,95] \rightarrow [70,90]$	66.667	40.396	193.75
$[70,90] \rightarrow [65,85]$	39.498	34.570	198.75
$[65,85] \rightarrow [70,90]$	39.364	29.374	210.00

TABLE V
Simulation Results - Level 2 - Chirp Test

	Error-Mean Square Value	**Error-Maximum Value**
$\nearrow$	42.465×10^{-6}	19.443×10^{-3}
$\searrow$	42.104×10^{-6}	19.860×10^{-3}

C. Level 3 Results

The third level considers three narrow band disturbances and is on this level where the advantages of the approach used for the central controller design are shown. Generally speaking, all the benchmark specifications were satisfied at this level, as can bee seen from the Tables I, VI and VII.

VII. Experimental Results

In this section the real-time results are presented. The same central controllers and adaptation gains used in simulation

TABLE I
SIMULATION RESULTS - SIMPLE STEP TEST

LEVEL 1							
Frequency (Hz)	**Global (dB)**	**Dist. Atte. (dB)**	**Max. Amp. (dB@Hz)**	**Norm^2 Trans.** ($\times 10^{-3}$)	**Norm^2 Res.** ($\times 10^{-3}$)	**Max. Val.** ($\times 10^{-3}$)	**Trans. (msec)**
50	34.17	44.43	7.21@67.18	14.478	3.751	17.675	248.750
55	32.78	46.84	6.66@78.12	10.146	4.432	19.895	88.750
60	32.30	46.94	6.76@82.81	10.226	4.685	20.136	73.750
65	32.77	48.38	6.95@50.00	9.296	4.512	19.905	52.500
70	33.29	50.04	7.84@53.13	8.314	4.300	19.939	41.250
75	34.01	51.90	7.31@53.13	7.923	4.048	19.817	45.000
80	34.57	51.56	7.65@93.75	8.161	3.750	21.749	43.750
85	34.39	52.29	6.70@64.06	9.753	3.697	23.902	75.000
90	31.94	44.98	7.46@68.75	13.335	4.037	26.842	121.250
95	23.87	35.70	6.17@87.50	20.764	4.550	29.124	305.00
LEVEL 2							
Frequency (Hz)	**Global (dB)**	**Dist. Atte. (dB)-(dB)**	**Max. Amp. (dB@Hz)**	**Norm^2 Trans.** ($\times 10^{-3}$)	**Norm^2 Res.** ($\times 10^{-3}$)	**Max. Val.** ($\times 10^{-3}$)	**Trans. (msec)**
50-70	39.16	41.81 - 47.49	6.11@79.69	25.035	4.258	29.441	171.25
55-75	38.35	48.33 - 47.66	7.74@87.50	23.392	4.747	34.729	136.25
60-80	39.42	50.14 - 49.26	7.87@50.00	19.405	4.170	34.186	90.00
65-85	39.83	49.13 - 51.04	7.24@53.13	18.266	3.941	36.547	97.50
70-90	38.45	51.49 - 42.59	7.61@59.37	27.326	4.238	40.645	198.750
75-95	36.47	54.02 - 37.28	6.06@87.50	35.621	4.061	44.513	197.50
LEVEL 3							
Frequency (Hz)	**Global (dB)**	**Dist. Atte. (dB)-(dB)-(dB)**	**Max. Amp. (dB@Hz)**	**Norm^2 Trans.** ($\times 10^{-3}$)	**Norm^2 Res.** ($\times 10^{-3}$)	**Max. Val.** ($\times 10^{-3}$)	**Trans. (msec)**
50-65-80	42.72	40.29 - 41.46 - 40.79	7.63@87.50	117.834	4.251	66.905	566.250
55-70-85	42.98	47.36 - 47.86 - 46.42	7.56@76.56	262.424	4.104	112.903	518.750
60-75-90	41.96	46.33 - 48.62 - 39.75	8.43@51.56	359.209	4.375	158.195	585.000
65-80-95	40.93	44.40 - 42.29 - 35.13	7.88@73.43	753.370	4.141	235.072	657.500

TABLE VI
SIMULATION RESULTS - LEVEL 3 - STEP CHANGES FREQUENCY TEST

SEQUENCE - 1			
Frequency (Hz)	**Norm^2 Trans.** ($\times 10^{-3}$)	**Max. Val.** ($\times 10^{-3}$)	**Transient (msec)**
$[55,70,85] \rightarrow [60,75,90]$	221.36	52.324	410.00
$[60,75,90] \rightarrow [55,70,85]$	170.29	55.00	398.75
$[55,70,85] \rightarrow [50,65,80]$	270.26	61.669	482.50
$[50,65,80] \rightarrow [55,70,85]$	195.22	63.247	338.75
SEQUENCE - 2			
$[60,75,90] \rightarrow [65,80,95]$	241.53	51.326	341.25
$[65,80,95] \rightarrow [60,75,90]$	187.52	67.680	338.75
$[60,75,90] \rightarrow [55,70,85]$	181.05	55.209	470.00
$[55,70,85] \rightarrow [60,75,90]$	198.20	51.033	343.75

TABLE VII
SIMULATION RESULTS - LEVEL 3 - CHIRP TEST

	Error-Mean Square Value	**Error-Maximum Value**
$\nearrow$	131.54×10^{-6}	44.252×10^{-3}
$\searrow$	118.3×10^{-6}	40.351×10^{-3}

have been considered in this section.

A. Level 1 Results

Table VIII presents the simple step test results. The proximity between the simulation and real-time results is remarkable. The main differences are in the maximum amplification and transient duration. For the step changes frequency test (Table IX) the results are quite similar and significant differences weren't found. In the chirp case, the results are also close to the simulation ones (Table X).

TABLE IX
EXPERIMENTAL RESULTS - LEVEL 1 - STEP CHANGES FREQUENCY TEST

SEQUENCE - 1			
Frequency (Hz)	**Norm^2 Trans.** ($\times 10^{-3}$)	**Max. Val.** ($\times 10^{-3}$)	**Transient (msec)**
$60 \rightarrow 70$	16.785	21.183	138.75
$70 \rightarrow 60$	16.803	19.259	122.5
$60 \rightarrow 50$	69.216	20.084	255
$50 \rightarrow 60$	46.052	33.437	170
SEQUENCE - 2			
Frequency (Hz)	**Norm^2 Trans.** ($\times 10^{-3}$)	**Max. Val.** ($\times 10^{-3}$)	**Transient (msec)**
$75 \rightarrow 85$	15.955	15.056	143.75
$85 \rightarrow 75$	15.528	18.733	152.5
$75 \rightarrow 65$	14.55	18.032	98.75
$65 \rightarrow 75$	14.223	18.733	130
SEQUENCE - 3			
Frequency (Hz)	**Norm^2 Trans.** ($\times 10^{-3}$)	**Max. Val.** ($\times 10^{-3}$)	**Transient (msec)**
$85 \rightarrow 95$	40.822	16.794	2.5
$95 \rightarrow 85$	30.727	23.647	168.75
$85 \rightarrow 75$	15.227	19.971	126.25
$75 \rightarrow 85$	16.044	14.343	150

TABLE X
EXPERIMENTAL RESULTS - LEVEL 1 - CHIRP TEST

	Error-Mean Square Value	**Error-Maximum Value**
$\nearrow$	13.742×10^{-6}	15.015×10^{-3}
$\searrow$	13.943×10^{-6}	15.015×10^{-3}

B. Level 2 Results

The similar behaviour is reflected in the Tables VIII, XI and XII, where the results are still close to the previous ones obtained in simulation.

TABLE VIII
EXPERIMENTAL RESULTS - SIMPLE STEP TEST

LEVEL 1							
Frequency (Hz)	**Global (dB)**	**Dist. Atte. (dB)**	**Max. Amp. (dB@Hz)**	**Norm^2 Trans.** ($\times 10^{-3}$)	**Norm^2 Res.** ($\times 10^{-3}$)	**Max. Val.** ($\times 10^{-3}$)	**Trans. (msec)**
50	34.60	38.49	9.83@65.63	14.478	4.88	13.86	392.50
55	34.54	50.45	9.48@118.75	13.32	4.94	19.97	103.75
60	33.34	49.49	8.23@79.69	14.72	5.16	21.19	81.25
65	32.78	50.04	9.65@90.63	14.17	4.53	20.50	120.00
70	30.54	47.90	9.01@89.06	14.73	4.87	22.96	113.75
75	29.53	45.54	8.90@50.00	11.20	4.86	19.28	57.50
80	30.28	48.72	8.49@95.31	8.14	4.17	21.14	43.750
85	28.47	45.94	10.66@57.81	10.05	6.90	25.14	22.50
90	28.02	42.65	8.24@73.44	17.08	6.94	25.11	110.00
95	24.63	34.55	9.06@82.81	50.09	8.33	32.44	616.25
LEVEL 2							
Frequency (Hz)	**Global (dB)**	**Dist. Atte. (dB)-(dB)**	**Max. Amp. (dB@Hz)**	**Norm^2 Trans.** ($\times 10^{-3}$)	**Norm^2 Res.** ($\times 10^{-3}$)	**Max. Val.** ($\times 10^{-3}$)	**Trans. (msec)**
50-70	34.42	33.58 - 42.90	8.32@59.38	32.10	9.35	29.02	393.75
55-75	33.27	44.90 - 44.18	11.85@115.63	32.49	8.04	30.23	212.50
60-80	33.42	45.59 - 41.70	7.78@118.75	31.35	7.08	28.99	230
65-85	31.72	40.01 - 43.66	8.02@106.25	22.75	7.62	31.44	112.50
70-90	32.91	41.43 - 38.63	7.52@59.38	21.38	6.05	33.90	192.50
75-95	31.04	48.89 - 34.66	7.09@87.50	28.33	6.65	38.40	222.50
LEVEL 3							
Frequency (Hz)	**Global (dB)**	**Dist. Atte. (dB)-(dB)-(dB)**	**Max. Amp. (dB@Hz)**	**Norm^2 Trans.** ($\times 10^{-3}$)	**Norm^2 Res.** ($\times 10^{-3}$)	**Max. Val.** ($\times 10^{-3}$)	**Trans. (msec)**
50-65-80	42.73	37.69 - 44.32 - 41.98	9.10@71.88	536.57	7.06	129.54	786.25
55-70-85	42.68	42.29 - 45.51 - 46.14	9.27@93.75	1,260.60	5.86	225.17	1,375
60-75-90	40.94	44.49 - 41.76 - 44.35	11.30@68.75	310.35	6.44	87.36	820
65-80-95	35.99	44.27 - 42.29 - 31.50	10.67@87.50	1,148.10	7.25	143.77	1,433.80

TABLE XI
EXPERIMENTAL RESULTS - LEVEL 2 - STEP CHANGES FREQUENCY TEST

SEQUENCE - 1			
Frequency (Hz)	**Norm^2 Trans.** ($\times 10^{-3}$)	**Max. Val.** ($\times 10^{-3}$)	**Transient (msec)**
$[55,75] \rightarrow [60,80]$	38.575	35.17	210
$[60,80] \rightarrow [55,75]$	38.506	31.007	160
$[55,75] \rightarrow [50,70]$	117.28	40.072	315
$[50,70] \rightarrow [55,75]$	69.186	43.262	157.5
SEQUENCE - 2			
Frequency (Hz)	**Norm^2 Trans.** ($\times 10^{-3}$)	**Max. Val.** ($\times 10^{-3}$)	**Transient (msec)**
$[70,90] \rightarrow [75,95]$	66.517	32.699	215
$[75,95] \rightarrow [70,90]$	60.013	33.827	205
$[70,90] \rightarrow [65,85]$	42.56	35.15	166.25
$[65,85] \rightarrow [70,90]$	42.946	28.575	216.25

TABLE XII
EXPERIMENTAL RESULTS - LEVEL 2 - CHIRP TEST

	Error-Mean Square Value	**Error-Maximum Value**
$\nearrow$	41.277×10^{-6}	21.624×10^{-3}
$\searrow$	42.379×10^{-6}	21.228×10^{-3}

TABLE XIII
EXPERIMENTAL RESULTS - LEVEL 3 - STEP CHANGES FREQUENCY TEST

SEQUENCE - 1			
Frequency (Hz)	**Norm^2 Trans.** ($\times 10^{-3}$)	**Max. Val.** ($\times 10^{-3}$)	**Transient (msec)**
$[55,70,85] \rightarrow [60,75,90]$	235.74	62.066	343.75
$[60,75,90] \rightarrow [55,70,85]$	208.75	55.581	278.75
$[55,70,85] \rightarrow [50,65,80]$	242.6	59.615	345
$[50,65,80] \rightarrow [55,70,85]$	235.63	76.772	282.5
SEQUENCE - 2			
$[60,75,90] \rightarrow [65,80,95]$	275.33	64.494	411.25
$[65,80,95] \rightarrow [60,75,90]$	225.24	56.829	400
$[60,75,90] \rightarrow [55,70,85]$	196.53	51.927	277.5
$[55,70,85] \rightarrow [60,75,90]$	183.17	54.69	350

TABLE XIV
EXPERIMENTAL RESULTS - LEVEL 3 - CHIRP TEST

	Error-Mean Square Value	**Error-Maximum Value**
$\nearrow$	158.15×10^{-6}	42.399×10^{-3}
$\searrow$	133.47×10^{-6}	52.203×10^{-3}

C. Level 3 Results

Finally, the most difficult level shows that the direct adaptive algorithm along with a central controller tuned with BSFs, presents a good performance regarding the benchmark specifications. This is confirmed in Tables VIII, XIII and XIV. From the simulation and real-time results, we can conclude that the frequency limits (50 and 95 Hz) are quite challenging due to the proximity of low damped complex zeros.

VIII. CONCLUDING REMARKS

Good coherence has been found between the simulation and real-time results, since the same controllers and adaptation gains were used on both situations. This shows the robustness of the scheme and the reliability of the model identified for the plant. The initial statements were confirmed since the proposed scheme shows fast transient durations, simpleness (the polynomial $\hat{Q}(q^{-1})$ is the only part estimated) and stability in closed loop. The computational complexity of the proposed algorithm is not very demanding.

REFERENCES

[1] F. Ben Amara, P.T. Kabamba, and A.G. Ulsoy. Adaptive sinusoidal disturbance rejection in linear discrete-time systems - Part I: Theory. *Journal of Dynamic Systems Measurement and Control*, 121:648–654, 1999.

[2] F. Ben Amara, P.T. Kabamba, and A.G. Ulsoy. Adaptive sinusoidal disturbance rejection in linear discrete-time systems - Part II: Experiments. *Journal of Dynamic Systems Measurement and Control*, 121:655–659, 1999.

[3] B.D.O. Anderson. From Youla-Kucera to identification, adaptive and nonlinear control. *Automatica*, 34:1485–1506, 1998.

[4] G. Bengtsson. Output regulation and internal models - a frequency domain approach. *Automatica*, 13(4):333 – 345, 1977.

[5] L.L. Beranek and I.L. Ver. *Noise and Vibration Control Engineering: Principles and Applications*. Wiley, New York, 1992.

[6] M. Bodson. Rejection of periodic distrubances of unknown and time-varying frequency. *Int. J. of Adapt. Contr. and Sign. Proc.*, 19:67–88, 2005.

[7] M. Bodson and S.C. Douglas. Adaptive algorithms for the rejection of sinusosidal disturbances with unknown frequency. *Automatica*, 33:2213–2221, 1997.

[8] Xu Chen and M. Tomizuka. A minimum parameter adaptive approach for rejecting multiple narrow-band disturbances with application to hard disk drives. *Control Systems Technology, IEEE Transactions on*, 20(2):408 –415, march 2012.

[9] Z. Ding. Global stabilization and disturbance suppression of a class of nonlinear systems with uncertain internal model. *Automatica*, 39(3):471 – 479, 2003.

[10] S.J. Elliott and P.A. Nelson. Active noise control. *Noise / News International*, pages 75–98, June 1994.

[11] S.J. Elliott and T.J. Sutton. Performance of feedforward and feedback systems for active control. *Speech and Audio Processing, IEEE Transactions on*, 4(3):214 –223, may 1996.

[12] G. Feng and M. Palaniswami. A stable adaptive implementation of the internal model principle. *IEEE Trans. on Automatic Control*, 37:1220–1225, 1992.

[13] B.A. Francis and W.M. Wonham. The internal model principle of control theory. *Automatica*, 12(5):457 – 465, 1976.

[14] C.R. Fuller and A.H. von Flotow. Active control of sound and vibration. *Control Systems, IEEE*, 15(6):9 –19, dec 1995.

[15] T. Gouraud, M. Gugliemi, and F. Auger. Design of robust and frequency adaptive controllers for harmonic disturbance rejection in a single-phase power network. *Proceedings of the European Control Conference, Bruxelles*, 1997.

[16] G. Hillerstrom and J. Sternby. Rejection of periodic disturbances with unknown period - a frequency domain approach. *Proceedings of American Control Conference, Baltimore*, pages 1626–1631, 1994.

[17] C.D. Johnson. Theory of disturbance-accomodating controllers. In *Control and Dynamical Systems* (C. T. Leondes, Ed.), 1976. Vol. 12, pp. 387-489.

[18] I. D. Landau, R. Lozano, M. M'Saad, and A. Karimi. *Adaptive control*. Springer, London, 2nd edition, 2011.

[19] I.D. Landau, T.B. Airimiţoaie, A. Castellanos Silva, and G. Buche. Benchmark on adaptive regulation. http://www.gipsa-lab.grenoble-inp.fr/~ioandore.landau/benchmark_adaptive_regulation/.

[20] I.D. Landau, T.B. Airimiţoaie, A. Castellanos Silva, and G. Buche. Benchmark on adaptive regulation: Rejection of unknown/time-varying multiple narrow band disturbances. In *Proceedings of the 12th biannual European Control Conference, Zürich, Suisse*, 2013.

[21] I.D. Landau, M. Alma, J.J. Martinez, and G. Buche. Adaptive suppression of multiple time-varying unknown vibrations using an inertial actuator. *Control Systems Technology, IEEE Transactions on*, 19(6):1327 –1338, nov. 2011.

[22] I.D. Landau, A. Constantinescu, and D. Rey. Adaptive narrow band disturbance rejection applied to an active suspension - an internal model principle approach. *Automatica*, 41(4):563–574, 2005.

[23] I.D. Landau and G. Zito. *Digital Control Systems - Design, Identification and Implementation*. Springer, London, 2005.

[24] R. Marino, G.L. Santosuosso, and P. Tomei. Robust adaptive compensation of biased sinusoidal disturbances with unknown frequency. *Automatica*, 39:1755–1761, 2003.

[25] Hynek Prochzka and Ioan Doré Landau. Pole placement with sensitivity function shaping using 2nd order digital notch filters. *Automatica*, 39(6):1103 – 1107, 2003.

[26] Z. Sun and T.C. Tsao. Adaptive control with asymptotic tracking performance and its application to an electro-hydraulic servo system. *Journal of Dynamic Systems Measurement and Control*, 122:188–195, 2000.

[27] Y.Z. Tsypkin. Stochastic discrete systems with internal models. *Journal of Automation and Information Sciences*, 29(4&5):156–161, 1997.

[28] S. Valentinotti. *Adaptive Rejection of Unstable Disturbances: Application to a Fed-Batch Fermentation*. Thèse de doctorat, École Polytechnique Fédérale de Lausanne, April 2001.

[29] B. Widrow and S.D. Stearns. *Adaptive Signal Processing*. Prentice-Hall, Englewood Cliffs, New Jersey, 1985.

[30] Y. Zhang, P.G. Mehta, R. Bitmead, and C.R. Johnson. Direct adaptive control for tonal disturbance rejection. *Proceedings of the American Control Conference, Philadelphia*, pages 1480–1482, 1998.