Adaptive feedforward compensation algorithms for AVC systems in presence of a feedback controller ☆

Marouane Alma[a], Ioan Doré Landau[a], Tudor-Bogdan Airimitoaie[a]

[a]GIPSA-LAB, Dept. of Automatic Control, ENSIEG BP 46, 38402 Saint-Martin d'Hères, France

Abstract

In [5] and [7] adaptation algorithms taking in account the "positive" feedback coupling arising in most of the active noise and vibration control systems have been proposed and analyzed. The stability of the system requires satisfaction of a positive real condition through an appropriate filtering of the regressor vector. It is shown in this note that the presence in addition of a feedback controller on one hand strongly influences the positive real conditions for stability and the structure of the filter to be used in the algorithm and on the other hand improves significantly the performance of the system. Experimental results obtained on an active vibration control (AVC) system clearly illustrate the benefit of using a hybrid adaptive feedforward + feedback approach.

Keywords: active vibration control, adaptive feedforward compensation, feedback control, adaptive control, parameter estimation.

1. Introduction

Adaptive feedforward for broadband disturbance compensation is widely used when a well correlated signal with the disturbance (image of the disturbance) is available ([2, 3, 6, 10]). However in many systems there is a positive (mechanical or acoustical) coupling between the feedforward compensation system and the measurement of the image of the disturbance.

In [5] and [7] adaptation algorithms taking in account this "positive" feedback have been proposed and analyzed. The stability of the system requires satisfaction of a positive real condition through an appropriate filtering of the regressor vector. The objective of this note is to show theoretically and experimentally what is the impact of using a feedback compensator in addition to an adaptive feedforward filter as discussed in [7].

Combination of adaptive feedforward + fixed feedback disturbance compensation has been already discussed since it is expected to improve the performance of active noise control (ANC) and active vibration control (AVC) systems. See for example [1, 9, 4]. However the influence of the feedback upon the stability of the adaptive feedforward algorithms has not been examined.

The main contributions of the present paper are:

- Establishing the influence of the feedback control loop upon the stability conditions for adaptive feedforward compensation (with and without internal positive coupling)

- Showing the improvement of the global attenuation w.r.t results obtained with adaptive feedforward compensation [7]

---

2. Basic Equations and Notations

The block diagram associated with an AVC system using an hybrid (feedback + adaptive feedforward) control is shown in figure 1.
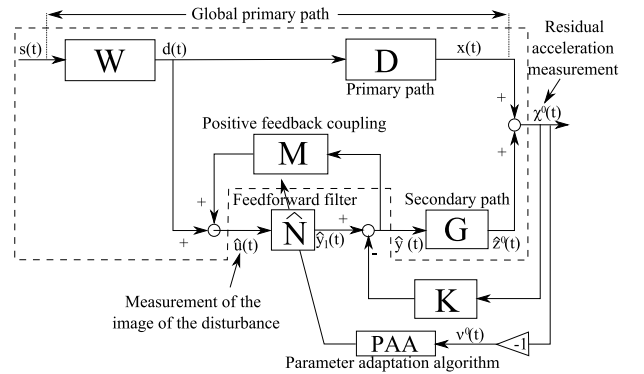

Figure 1: Feedforwrd AVC with fixed feedback controller (K) and adaptive feedforward compensator ($\hat{N}$).

The description, equations and notations of the various blocs and transfer functions have been presented in detail in [7] eqs. (1) to (12). $D = \frac{B_D}{A_D}, G = \frac{B_G}{A_G}, M = \frac{B_M}{A_M}$ represent the transfer operators associated with the primary, secondary and reverse paths (all asymptotically stable). The feedforward compensator is $\hat{N} = \frac{\hat{R}}{\hat{S}}$ with:

$$\hat{R}(q^{-1}) = \hat{r}_0 + \hat{r}_1 q^{-1} + \ldots + \hat{r}_{n_{\hat{R}}} q^{-n_{\hat{R}}}, \quad (1)$$

$$\hat{S}(q^{-1}) = 1 + \hat{s}_1 q^{-1} + \ldots + \hat{s}_{n_{\hat{S}}} q^{-n_{\hat{S}}} = 1 + q^{-1}\hat{S}^*(q^{-1}). \quad (2)$$

The signal $s(t)$ is the external disturbance source, $d(t)$ is the correlated disturbance measurement (in the absence of the compensation) and $\hat{u}(t)$ is the measured primary signal which

is the sum of $d(t)$ and of the effect of the actuator used for compensation.

The fixed feedback controller K, is characterized by the stable transfer function:

$$K(q^{-1}) = \frac{B_K(q^{-1})}{A_K(q^{-1})} = \frac{b_0^K + b_1^K q^{-1} + ... + b_{n_{B_K}}^K q^{-n_{B_K}}}{1 + a_1^K q^{-1} + ... + a_{n_{A_K}}^K q^{-n_{A_K}}}. \quad (3)$$

The "a posteriori" output of the feedforward filter is denoted by: $\hat{y}_1(t+1) = \hat{y}_1(t+1|\hat{\theta}(t+1))$.

The "a priori" output of the estimated feedforward filter is given by:

$$\begin{aligned} \hat{y}_1^0(t+1) &= \hat{y}_1(t+1|\hat{\theta}(t)) \\ &= \hat{\theta}^T(t)\phi(t) = [\hat{\theta}_S^T(t), \hat{\theta}_R^T(t)] \begin{bmatrix} \phi_{\hat{y}_1}(t) \\ \phi_{\hat{u}}(t) \end{bmatrix} \end{aligned} \quad (4)$$

where

$$\hat{\theta}^T(t) = [\hat{s}_1(t)...\hat{s}_{n_S}(t), \hat{r}_0(t)...\hat{r}_{n_R}(t)] = [\hat{\theta}_S^T(t), \hat{\theta}_R^T(t)] \quad (5)$$

$$\begin{aligned} \phi^T(t) &= [-\hat{y}_1(t)... - \hat{y}_1(t-n_S+1), \hat{u}(t+1), \hat{u}(t)...\hat{u}(t-n_R+1)] \\ &= [\phi_{\hat{y}_1}^T(t), \phi_{\hat{u}}^T(t)] \end{aligned} \quad (6)$$

and $\hat{y}_1(t), \hat{y}_1(t-1)$ ... are the "a posteriori" outputs of the feedforward filter generated by:

$$\hat{y}_1(t+1) = \hat{y}_1(t+1|\hat{\theta}(t+1)) = \hat{\theta}^T(t+1)\phi(t) \quad (7)$$

while $\hat{u}(t+1)$, $\hat{u}(t)$... are the measurements provided by the primary transducer[1].

The control signal applied to the secondary path is given by

$$\hat{y}(t+1) = \hat{y}_1(t+1) - \frac{B_K}{A_K}\chi^0(t+1) \quad (8)$$

where $\chi^0(t+1)$ is the measured residual acceleration.

3. Development of the Algorithms

The algorithms for adaptive feedforward compensation in presence of feedback controller will be developed under the hypothesis H1, H3 and H4 from [7] and new hypothesis H2:

H2 - Perfect matching condition. There exists a filter $N(q^{-1})$ of finite dimension such that[2]:

$$\frac{N}{(1-NM)}G = -D \quad (9)$$

and the characteristic polynomials (i) of the "internal" positive coupling loop:

$$P = A_M S - B_M R \quad (10)$$

(ii) of the closed loop (G-K):

$$P_{cl} = A_G A_K + B_G B_K \quad (11)$$

and of the coupled feedforward-feedback loop:

$$P_{fb-ff} = A_M S[A_G A_K + B_G B_K] - B_M R A_K A_G \quad (12)$$

are Hurwitz polynomials.

A first step in the development of the algorithms is to establish a relation between the errors on the estimation of the parameters of the feedforward filter and the measured residual acceleration. This is summarized in the following lemma.

Lemma 3.1: *Under hypotheses H1, H2, H3 and H4, for the system described in Section 2, using a feedforward compensator $\hat{N}$ with constant parameters and a feedback controller K, one has:*

$$\nu(t+1) = \frac{A_M A_G A_K G}{P_{fb-ff}}[\theta - \hat{\theta}]^T\phi(t) \quad (13)$$

where

$$\theta^T = [s_1,...s_{n_S}, r_0, r_1,...r_{n_R}] = [\theta_S^T, \theta_R^T] \quad (14)$$

is the vector of parameters of the optimal filter N assuring perfect matching

$$\hat{\theta}^T = [\hat{s}_1...\hat{s}_{n_S}, \hat{r}_0...\hat{r}_{n_R}] = [\hat{\theta}_S^T, \hat{\theta}_R^T] \quad (15)$$

is the vector of constant estimated parameters of $\hat{N}$

$$\begin{aligned} \phi^T(t) &= [-\hat{y}_1(t)... - \hat{y}_1(t-n_S+1), \hat{u}(t+1), \hat{u}(t)...\hat{u}(t-n_R+1)] \\ &= [\phi_{\hat{y}_1}^T(t), \phi_{\hat{u}}^T(t)] \end{aligned} \quad (16)$$

and $\hat{u}(t+1)$ is given by:[3]

$$\hat{u}(t+1) = d(t+1) + \frac{B_M^*}{A_M}\hat{y}(t). \quad (17)$$

The proof is given in the Appendix.

Corollary 1: *For $B_K = 0$ (absence of the feedback controller), the error equation for pure feedforward compensation given in [7], is obtained.*

Corollary 2: *For $B_M = 0$ (absence of the mechanical coupling), the error equation is given by:*

$$\nu(t+1) = \frac{B_G A_K}{P_{cl}S}[\theta - \hat{\theta}]^T\phi(t) = \frac{G_{cl}}{S}[\theta - \hat{\theta}]^T\phi(t) \quad (18)$$

where: G_{cl} is the closed loop transfer function (G,K) defined by: $G_{cl} = \frac{B_G A_K}{P_{cl}}$.

Filtering the vector $\phi(t)$ through an asymptotically stable filter $L(q^{-1}) = \frac{B_L}{A_L}$, equation (13) for $\hat{\theta} = $ constant becomes:

$$\nu(t+1) = \frac{A_M A_G A_K G}{P_{fb-ff}L}[\theta - \hat{\theta}]^T\phi_f(t) \quad (19)$$

$$\phi_f(t) = L(q^{-1})\phi(t). \quad (20)$$

Equation (19) will be used to develop the adaptation algorithms neglecting the non-commutativity of the operators when $\hat{\theta}$ is time varying (however an exact algorithm can be derived in such cases - see [8]).

---

[1] $\hat{u}(t+1)$ is available before adaptation of parameters starts at $t+1$
[2] In many cases, the argument q^{-1} or z^{-1} will be dropped out

[3] $B(q^{-1}) = q^{-1}B^*(q^{-1})$

Replacing the fixed estimated parameters by the current estimated parameters, equation (19) becomes the equation of the a-posteriori residual (adaptation) error $v(t+1)$ (which is computed):

$$v(t+1/\hat{\theta}(t+1)) = \frac{A_M A_G A_K}{P_{fb-ff}L} G[\theta - \hat{\theta}(t+1)]^T \phi_f(t). \quad (21)$$

Equation (21) has the standard form for an a-posteriori adaptation error ([8]), which immediately suggests to use the following parameter adaptation algorithm (same as in [7]):

$$\hat{\theta}(t+1) = \hat{\theta}(t) + F(t)\psi(t)v(t+1) ; \quad (22)$$

$$v(t+1) = \frac{v^0(t+1)}{1 + \psi^T(t)F(t)\psi(t)} ; \quad (23)$$

$$F(t+1) = \frac{1}{\lambda_1(t)} \left[F(t) - \frac{F(t)\psi(t)\psi^T(t)F(t)}{\frac{\lambda_1(t)}{\lambda_2(t)} + \psi^T(t)F(t)\psi(t)} \right] \quad (24)$$

$$1 \geq \lambda_1(t) > 0; 0 \leq \lambda_2(t) < 2; F(0) > 0 \quad (25)$$

$$\psi(t) = \phi_f(t) \quad (26)$$

where $\lambda_1(t)$ and $\lambda_2(t)$ allow to obtain various profiles for the matrix adaptation gain $F(t)$ (see section 4 and [8]).

Three choices for the filter L will be considered, leading to three different algorithms:

Algorithm *I*: $L = G$
Algorithm *II*: $L = \hat{G}$
Algorithm *III*: $L = \frac{\hat{A}_M \hat{A}_G A_K}{\hat{P}_{fb-ff}} \hat{G}$
where:

$$\hat{P}_{fb-ff} = \hat{A}_M \hat{S}[\hat{A}_G A_K + \hat{B}_G B_K] - \hat{B}_M \hat{R} A_K \hat{A}_G \quad (27)$$

is an estimation of the characteristic polynomial of the coupled feedforward-feedback loop computed on the basis of available estimates of the parameters of the filter $\hat{N}$ and estimated models $\hat{G} = \frac{\hat{B}_G}{\hat{A}_G}$ and $\hat{M} = \frac{\hat{B}_M}{\hat{A}_M}$. For the Algorithm *III* several options for updating $\hat{P}_{fb-ff}$ can be considered:

- Run Algorithm *II* for a certain time to get estimates of $\hat{R}$ and $\hat{S}$ and compute $\hat{P}_{fb-ff}$

- Update $\hat{P}_{fb-ff}$ at each sampling instant or from time to time using Algorithm *III* (after a short initialization horizon using Algorithm *II*)

3.1. Analysis of the Algorithms

For Algorithms *I*, *II* and *III* the equation for the a-posteriori adaptation error has the form:[4]

$$v(t+1) = H(q^{-1})[\theta - \hat{\theta}(t+1)]^T \psi(t) \quad (28)$$

where:

$$H(q^{-1}) = \frac{A_M A_G A_K}{P_{fb-ff}L} G, \; \psi = \phi_f. \quad (29)$$

Neglecting the non-commutativity of time varying operators, one has the following result:

---

[4]the argument $\hat{\theta}(t+1)$ has been dropped out

Lemma 3.2: *Assuming that eq. (28) represents the evolution of the a posteriori adaptation error and that the parameter adaptation algorithm (22) through (26) is used, one has*:

$$\lim_{t\to\infty} v(t+1) = 0 \quad (30)$$

$$\lim_{t\to\infty} \frac{[v^0(t+1)^2]}{1 + \psi(t)^T F(t)\psi(t)} = 0 \quad (31)$$

$$||\psi(t)|| \text{ is bounded} \quad (32)$$

$$\lim_{t\to\infty} v^0(t+1) = 0 \quad (33)$$

for any initial conditions $\hat{\theta}(0), v^0(0), F(0)$, provided that:

$$H'(z^{-1}) = H(z^{-1}) - \frac{\lambda_2}{2}, \max_t [\lambda_2(t)] \leq \lambda_2 < 2 \quad (34)$$

is a strictly positive real (SPR) transfer function.
The proof is similar to that given in [7] for $B_K = 0$ and $A_K = 1$ (absence of the feedback controller) and it is omitted.

4. Experimental results

The same AVC system as in [7] has been used.

4.1. Design of the feedback controller

The objective of the feedback controller K is to reduce the disturbance effect on the residual acceleration $\chi(t)$ where the secondary path G has enough gain, without using the disturbance correlated measurement $\hat{u}(t)$.

4.2. Broadband disturbance rejection

The adaptive feedforward filter structure for most of the experiments has been $n_R = 9$, $n_S = 10$ (total of 20 parameters) and this complexity does not allow to verify the "perfect matching condition" (which requires more than 40 parameters). A pseudo-random binary sequence (PRBS) excitation on the global primary path will be considered as the disturbance. For the *adaptive* operation the Algorithms *II* and *III* have been used with decreasing adaptation gain ($\lambda_1(t) = 1$, $\lambda_2(t) = 1$) combined with a *constant trace* adaptation gain.

The experiments have been carried on by first applying the disturbance in open loop during 50*s* and after that closing the loop with the hybrid adaptive feedforward-feedback algorithms. Time domain results obtained in open loop and with hybrid control (using Algorithm *III*) on the AVC system are shown in figure 2. The initial trace of the matrix adaptation gain was 10 and the constant trace has been fixed at 0.2.

Table 1 summarizes the global attenuation results for various configurations. Clearly, hybrid adaptive feedforward-feedback scheme brings a significant improvement in performance with respect to adaptive feedforward compensation alone. Comparing with the results of [7], (Table 2) one can conclude that in terms of performance and complexity it is more interesting to add a linear feedback than augmenting the number of parameters of the adaptive feedforward filter beyond a certain value.
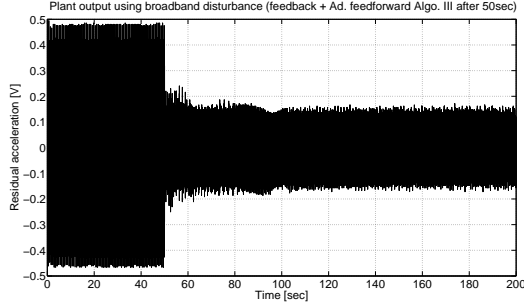
Figure 2: Real time results obtained with feedback controller and adaptive feedforward Algorithm *III*.

	No feedback no feedforward	Feedback only	Adaptive feedforward	Feedback & Ad. feedforward
Variance	0.0354	0.0067	0.0054	0.0033
Normalized var.	1	0.1892	0.1525	0.0932
Atten. (dB)	0dB	-14.40dB	-16.23dB	-20.53dB

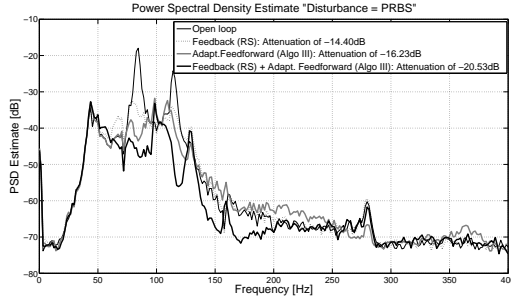Table 1: Global attenuation for various configurations.

Figure 3: Power spectral densities of the residual acceleration (Disturbance = PRBS).

Figure 3 shows a comparison of the power spectral densities for adaptive feedforward alone with 20 parameters (Alg. *III*), feedback controller alone, and the hybrid "feedback-adaptive feedforward" scheme with 20 parameters (Alg. *III*).

5. Conclusions

The theoretical analysis presented in this note, has pointed out the interaction between the feedback and the stability conditions for adaptive feedforward compensation. Experimental results on an AVC system featuring an internal "positive" coupling have illustrated the improvement in the performance provided by the hybrid approach.

6. Appendix- Proof of lemma 3.1

For a fixed value of the parameter vector $\hat{\theta}$ characterizing the estimated filter $\hat{N}(q^{-1})$ of same dimension as the optimal filter $N(q^{-1})$, the output of the secondary path can be expressed by (in this case $\hat{z}(t) = \hat{z}^0(t)$, $\hat{y}(t) = \hat{y}^0(t)$ and $\chi(t) = \chi^0(t)$):

$$\hat{z}(t) = G\hat{y}(t) \qquad (35)$$

with:

$$\hat{y}(t) = \hat{y}_1(t) - \frac{B_K}{A_K}\chi(t) = \hat{y}_1(t) + \frac{B_K}{A_K}v(t) \qquad (36)$$

where:

$$\hat{y}_1(t+1) = \hat{\theta}^T\phi(t). \qquad (37)$$

The key observation is that using equations (63) through (67) from [7] the dummy variable y(t+1) can be expressed as :

$$y(t+1) = \theta^T\phi(t) - S^*[y(t) - \hat{y}_1(t)] + R[u(t+1) - \hat{u}(t+1)]. \qquad (38)$$

Define the dummy error (for a fixed vector $\hat{\theta}$)

$$\varepsilon(t+1) \quad = \quad y(t+1) - \hat{y}_1(t+1) - KG\varepsilon(t+1) \qquad (39)$$

and the residual error becomes:

$$v(t+1) = -x(t+1) - \hat{z}(t+1) = G\varepsilon(t+1). \qquad (40)$$

By taking into account the equations 36 and 40, $y(t+1)$ becomes:

$$y(t+1) = \theta^T\phi(t) - S^*[y(t) - \hat{y}(t) + \frac{B_KB_G}{A_KA_G}\varepsilon(t)]$$
$$+ R[u(t+1) - \hat{u}(t+1)]. \qquad (41)$$

It results from (41) by taking into account the expressions of $u(t)$ and $\hat{u}(t)$ given by (67) of [7] and (17) that:

$$y(t+1) = \theta^T\phi(t) - \left[S^*(1 + \frac{B_KB_G}{A_KA_G}) - \frac{R(q^{-1})B_M^*}{A_M}\right]\varepsilon(t). \qquad (42)$$

Using equations (36) and (39), one gets (after passing all terms in ε on the left hand side):

$$\varepsilon(t+1) = \frac{A_MA_GA_K}{P_{fb-ff}}[\theta - \hat{\theta}]^T\phi(t). \qquad (43)$$

Taking now into account equation (40) one obtains equation (13). End of the proof.

References

[1] R. De Callafon. Active noise control method and apparatus including feedforward and feedback controller, March 2010.

[2] S.J. Elliott and P.A. Nelson. Active noise control. *Noise / News International*, pages 75–98, June 1994.

[3] S.J. Elliott and T.J. Sutton. Performance of feedforward and feedback systems for active control. *IEEE Transactions on Speech and Audio Processing*, 4(3):214–223, May 1996.

[4] E. Esmailzadeh, A. Alasty, and A.R. Ohadi. Hybrid active noise control of a one-dimensional acoustic duct. *Journal of vibration and acoustics*, 124(1):10–18, 2002.

[5] C.A Jacobson, C.R Johnson, D.C Mc Cormick, and W.A Sethares. Stability of active noise control algorithms. *IEEE Signal Processing letters*, 8(3):74–76, 2001.

[6] M.S. Kuo and D.R. Morgan. Active noise control: A tutorial review. *Proceedings of the IEEE*, 87:943–973, 1999.

[7] I.D. Landau, M. Alma, and T.B. Airimiţoaie. Adaptive feedforward compensation algorithms for active vibration control with mechanical coupling. *Automatica*, 47(10):2185 – 2196, 2011.

[8] I.D. Landau, R. Lozano, and M. M'Saad. *Adaptive control*. Springer, London, 1997.

[9] L.R. Ray, J.A. Solbeck, A.D. Streeter, and R.D. Collier. Hybrid feedforward-feedback active noise reduction for hearing protection and communication. *The Journal of the Acoustical Society of America*, 120(4):2026–2036, 2006.

[10] J Zeng and R.A de Callafon. Recursive filter estimation for feedforward noise cancellation with acoustic coupling. *Journal of sound and vibration*, 291:1061–1079, 2006.