Improving adaptive feedforward vibration compensation by using "Integral + Proportional" adaptation☆

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Abstract

Analysis of various adaptive feedforward vibration compensation schemes has shown that a strictly positive real (SPR) condition has to be satisfied in order to guarantee the stability of the whole system [1–4]. Filters have to be implemented in order to satisfy this condition. The problem becomes even more crucial in the presence of the internal mechanical coupling between the compensator system and the reference source (a correlated measurement with the disturbance) since some information is not available when adaptation starts (see [3]). It is therefore very important to relax the SPR condition at least in the initial phase and in the same time to improve the adaptation transients. It is shown, in this paper, that adding a proportional adaptation to the standard integral type parametric adaptation, the SPR condition can be relaxed and the adaptation transients are improved. Theoretical developments are enhanced by real time experimental results obtained on an active vibration control (AVC) system.

Keywords: active vibration control, adaptive feedforward compensation, adaptive control, stability, parameter estimation

1. Introduction

An important issue in adaptive feedforward compensation is the design of filters either on the observed variables of the feedforward compensator ([3]) or on the residual acceleration ([5]) in order to satisfy positive realness conditions on some transfer functions required by the stability analysis. The problem becomes even more crucial in the presence of the internal mechanical coupling between the compensator system and the reference source (a correlated measurement with the disturbance) since some information is not available when adaptation starts (see [3]). In [3], based on work done in [6], it was shown that for small adaptation gains (slow adaptation) violation of the SPR condition in some frequency regions is acceptable provided that in the average the input-output product associated with this transfer function is positive. However, the performances are degraded with respect to the case when the SPR condition is satisfied. It is in fact a signal dependent condition.

The problem of removing or relaxing the positive real condition can be also approached by adding a proportional adaptation to the widely used integral adaptation. While this approach is known in adaptive control ([7, 8]) apparently it has not been used in the context of adaptive feedforward compensation. Furthermore, it was observed in adaptive control, that adding positive proportional adaptation will speed up the adaptation transients ([8, 9]).

"Integral + Proportional" (IP) adaptation has been discussed from a stability point of view in [8, 9] for the case of constant integral adaptation gain. A stable IP adaptation with time varying integral adaptation gain has been introduced in [10]. The objective of this paper is to explore the advantages of adding proportional adaptation in the context of the adaptive feedforward compensation of vibrations both from the theoretical and applications points of view.

The main contributions of the present paper are: (i) development and stability analysis of IP adaptation algorithms for adaptive feedforward compensation in the presence (or not) of an internal positive feedback, (ii) relaxation of the SPR condition in the context of adaptive feedforward compensation using IP adaptation, and (iii) application of the IP adaptation algorithms to an AVC system featuring internal positive mechanical coupling and comparison with existing algorithms.

2. Development and Analysis of the Algorithms

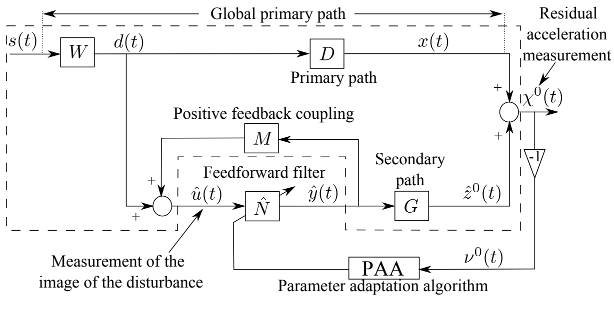

Figure 1: Feedforward AVC with adaptive feedforward compensator.

The block diagram of the adaptive feedforward compensator in the presence of an internal positive feedback associated

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with an AVC system is shown in Fig. (1). $D = \frac{B_D}{A_D}$, $G = \frac{B_G}{A_G}$, and $M = \frac{B_M}{A_M}$ represent the transfer operators associated with the primary, secondary, and reverse (positive coupling) paths.

The optimal feedforward filter (unknown) is defined by $N(q^{-1}) = \frac{R(q^{-1})}{S(q^{-1})} = \frac{r_0+\ldots+r_{n_R}q^{-n_R}}{1+s_1q^{-1}+\ldots+s_{n_S}q^{-n_S}}$. The estimated filter will be denoted by $\hat{N}(t,q^{-1})$ during estimation (adaptation) of its parameters and by $\hat{N}(q^{-1})$ when it is a linear filter with constant coefficients.

The parameters of the optimal filter and of the estimated filter are respectively given by the vectors

$$\theta^T = [s_1,\ldots s_{n_S},\ r_0,\ldots r_{n_R}] = [\theta_S^T,\ \theta_R^T], \tag{1}$$

$$\hat{\theta}^T(t) = [\hat{s}_1(t),\ldots \hat{s}_{n_S}(t),\ \hat{r}_0(t),\ldots \hat{r}_{n_R}(t)] = [\hat{\theta}_S^T(t),\ \hat{\theta}_R^T(t)], \tag{2}$$

while the observations vector is given by

$$\begin{aligned}\phi^T(t) &= [-\hat{y}(t),\ldots -\hat{y}(t-n_S+1),\ \hat{u}(t+1),\ldots \hat{u}(t-n_R+1)]\\ &= [\phi_{\hat{y}}^T(t),\ \phi_{\hat{u}}^T(t)],\end{aligned} \tag{3}$$

where $\hat{u}(t)$ and $\hat{y}(t)$ are the input and output of the estimated feedforward filter. $\hat{u}(t)$ is a measurable signal satisfying (see Fig. (1))

$$\hat{u}(t+1) = d(t+1) + \frac{B_M^*(q^{-1})}{A_M(q^{-1})}\hat{y}(t) \tag{4}$$

and $\hat{y}(t)$ denotes the *a posteriori* output

$$\hat{y}(t+1) = \hat{y}(t+1|\hat{\theta}(t+1)) = \hat{\theta}^T(t+1)\phi(t). \tag{5}$$

Similarly to (5), the *a priori* output is obtained by taking the parameters' estimates from the previous time step: $\hat{y}^0(t+1) = \hat{y}(t+1|\hat{\theta}(t))$.

The measurable residual error satisfies

$$\chi^0(t+1) = \chi(t+1|\hat{\theta}(t)) = \hat{z}^0(t+1) + x(t+1), \tag{6}$$

where $\hat{z}^0(t)$ is the output of the secondary path and $x(t)$ is the output of the primary path. $\hat{z}^0(t)$ and $x(t)$ can not be measured when the feedforward compensator is active. The previous equation can be used to immediately obtain the *a priori* adaptation error as

$$\nu^0(t+1) = -\chi^0(t+1) = -x(t+1) - \hat{z}^0(t+1). \tag{7}$$

Using Lemma 4.1 and eqs. (26) through (30) from [3], it results that the *a posteriori* adaptation error, $\nu(t+1) = \nu(t+1|\hat{\theta}(t+1))$, which is computed, satisfies:

$$\nu(t+1) = \frac{A_M(q^{-1})G(q^{-1})}{P(q^{-1})L(q^{-1})}[\theta - \hat{\theta}(t+1)]^T\phi_f(t), \tag{8}$$

where

$$\phi_f(t) = L(q^{-1})\phi(t). \tag{9}$$

Eq. (8) has the standard form for an *a posteriori* adaptation error ([8]). The following "Integral + Proportional" parameter adaptation algorithm (IP-PAA) is proposed:

$$\hat{\theta}_I(t+1) = \hat{\theta}_I(t) + \xi(t)F_I(t)\Phi(t)\nu(t+1), \tag{10a}$$

$$\hat{\theta}_P(t+1) = F_P(t)\Phi(t)\nu(t+1), \tag{10b}$$

$$\nu(t+1) = \frac{\nu^0(t+1)}{1+\Phi^T(t)(\xi(t)F_I(t)+F_P(t))\Phi(t)}, \tag{10c}$$

$$F_I(t+1) = \frac{1}{\lambda_1(t)}\left[F_I(t) - \frac{F_I(t)\Phi(t)\Phi^T(t)F_I(t)}{\frac{\lambda_1(t)}{\lambda_2(t)} + \Phi^T(t)F_I(t)\Phi(t)}\right], \tag{10d}$$

$$F_P(t) = \alpha(t)F_I(t);\ \alpha(t) > -0.5, \tag{10e}$$

$$F(t) = \xi(t)F_I(t) + F_P(t), \tag{10f}$$

$$\xi(t) = 1 + \frac{\lambda_2(t)}{\lambda_1(t)}\Phi^T(t)F_P(t)\Phi(t), \tag{10g}$$

$$\hat{\theta}(t+1) = \hat{\theta}_I(t+1) + \hat{\theta}_P(t+1), \tag{10h}$$

$$0 < \lambda_1(t) \leq 1,\ 0 \leq \lambda_2(t) < 2,\ F_I(0) > 0, \tag{10i}$$

$$\Phi(t) = \phi_f(t), \tag{10j}$$

where $\nu(t+1)$ is the filtered adaptation error, $\lambda_1(t)$ and $\lambda_2(t)$ allow to obtain various profiles for the matrix adaptation gain $F_I(t)$ (see [8] for more details). By taking $\lambda_2(t) \equiv 0$ one obtains a constant adaptation gain matrix (and choosing $F_I = \gamma I$, $\gamma > 0$ one gets a scalar adaptation gain). For $\alpha(t) \equiv 0$, one obtains the algorithm with integral adaptation gain introduced in [3][1].

Three choices for the filter L will be considered, leading to three different algorithms:

$$\begin{aligned}&\text{Algorithm } I && L = G\\ &\text{Algorithm } II && L = \hat{G}\\ &\text{Algorithm } III && L = \frac{\hat{A}_M}{\hat{P}}\hat{G}\end{aligned} \tag{11}$$

where

$$\hat{P} = \hat{A}_M\hat{S} - \hat{B}_M\hat{R} \tag{12}$$

is an estimation of the characteristic polynomial of the internal feedback loop computed on the basis of available estimates for the parameters of the filter $\hat{N}$. To use Algorithm *III* one has to start with Algorithm *II* where the SPR condition is in general not satisfied. Therefore, relaxing the SPR condition for Algorithm *II* is very important.

2.1. Analysis of the Algorithms

For Algorithms *I*, *II*, and *III*, using (8) and (9), the equation for the *a posteriori* adaptation error has the form

$$\nu(t+1) = H(q^{-1})[\theta - \hat{\theta}(t+1)]^T\Phi(t), \tag{13}$$

where

$$H(q^{-1}) = \frac{A_M(q^{-1})G(q^{-1})}{P(q^{-1})L(q^{-1})},\ \Phi = \phi_f = L(q^{-1})\phi. \tag{14}$$

[1] The present algorithm is a generalization of the algorithms given in columns 1 and 2 of Table 1 of [3].

Thus for Algorithm *II* one has $H(q^{-1}) = \frac{A_M G}{P\hat{G}}$ and for Algorithm *III* one obtains $H(q^{-1}) = \frac{A_M G\hat{P}}{P\hat{A}_M\hat{G}}$. Note that in this last case the SPR condition is always satisfied provided that one has good estimations of A_M, G, and P (for more details see [3]).

Neglecting the non-commutativity of time varying operators, one has the following result:

Lemma 2.1. *Assuming that eq.* (13) *represents the evolution of the a posteriori adaptation error and that the IP-PAA* (10) *is used, one has:*

$$\lim_{t\to\infty} \nu(t+1) = 0, \tag{15}$$

$$\lim_{t\to\infty} \frac{[\nu^0(t+1)^2]}{1+\Phi(t)^T F(t)\Phi(t)} = 0, \tag{16}$$

$$||\Phi(t)|| \text{ is bounded}, \tag{17}$$

$$\lim_{t\to\infty} \nu^0(t+1) = 0, \tag{18}$$

for any initial conditions $\hat{\theta}(0), \nu^0(0), F(0)$*, provided that*

$$H'(q^{-1}) = H(q^{-1}) - \frac{\lambda_2}{2}, \ \max_t \lambda_2(t) \le \lambda_2 < 2. \tag{19}$$

is a SPR transfer function.

The proof[2] of (15) is given in Appendix A. For (16), (17), and (18), the proof follows [3, 11] and it is omitted. The proof of [10] for IP adaptation with time varying integral adaptation gain is given for $\xi(t) = \frac{1}{\lambda_1(t)} + \frac{\lambda_2(t)}{\lambda_1(t)}\Phi^T(t)F_P(t)\Phi(t)$. To the knowledge of the authors, the proof for $\xi(t)$ given in eq. (10g) is presented here for the first time.

3. Relaxing the Positive Real Condition

An equivalent feedback system can be associated to the IP-PAA where the feedforward path is characterized by the transfer function $H'(z^{-1})$. There is additional "excess" of passivity in the feedback path (that depends upon the adaptation gains and the magnitude of $\Phi(t)$) which can be transferred to the linear feedforward block in order to relax the SPR condition. This idea was prompted out in the context of recursive identification by Tomizuka and results have been given for the case of integral adaptation and for the case when the equivalent linear feedforward path is characterized by an all poles (no zeros) transfer function (see [7]). These results have been extended in [8] for IP adaptation with constant adaptation gain. Taking into account the poles-zeros structure of $H(q^{-1})$, the results of [7, 8] should be extended for the situation described in this paper. One has the following result:

Lemma 3.1. *Given the discrete transfer function*

$$H(z^{-1}) = \frac{B(z^{-1})}{A(z^{-1})} = \frac{b_0 + b_1 z^{-1} + \ldots + b_{n_B} z^{-n_B}}{1 + a_1 z^{-1} + \ldots + a_{n_A} z^{-n_A}}, \tag{20}$$

under the hypotheses:

[2] $\nu^0(t+1)$ is computed using $\hat{\theta}(t) = \hat{\theta}_I(t)$.

H1) $H(z^{-1})$ *has all its zeros inside the unit circle,*

H2) $b_0 \ne 0$,

there exists a positive scalar gain K such that $\frac{H}{1+KH}$ *is SPR.*

The proof of this lemma is presented in Appendix B.

Using the above property, for the IP-PAA given by the eqs. (10) and eq. (13) for $\lambda_2(t) \equiv 0$, $\lambda_1(t) \equiv 1$ (constant adaptation gain), and choosing K such that $\frac{H}{1+KH}$ is SPR, one gets the equivalent feedback system:

$$\nu(t+1) = -\frac{H(z^{-1})}{1+KH(z^{-1})} y_{e2}(t), \tag{21}$$

$$\tilde{\theta}_I(t+1) = \tilde{\theta}_I(t) + \xi(t)F_I\Phi(t)\nu(t+1), \tag{22}$$

$$\tilde{\theta}_I(t) = \hat{\theta}_I(t) - \theta, \tag{23}$$

$$y_{e2}(t) = \Phi^T(t)\tilde{\theta}_I(t) + (\Phi^T(t)F(t)\Phi(t) + K)\nu(t+1). \tag{24}$$

Fig. 2 shows the equivalent feedback system associated with the I+P adaptation algorithm of eqs. (10) and eq. (13). The introduction of the scalar gain K does not change the characteristics of the feedback loop but it allows to show how passivity can be passed from the feedback path to the feedforward path.

To prove the stability it remains to show that the new feedback path given by (24) is weakly passive, *i.e.*, it satisfies the Popov inequality

$$\sum_{t=0}^{t_1} y_{e2}(t)u_{e2}(t) \ge -\gamma_0^2. \tag{25}$$

The following theorem provides the necessary result.

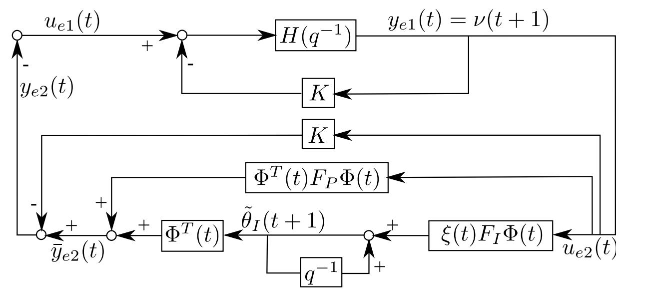

Figure 2: Equivalent feedback representation of the PAA with "Integral + Proportional" adaptation and scalar gain K.

Theorem 3.1. *The adaptive system described by eq.* (8) *and eqs.* (10) *for* $\lambda_2(t) \equiv 0$ *and* $\lambda_1(t) \equiv 1$ *is asymptotically stable provided that:*

T1) It exists a gain K such that $\frac{H}{1+KH}$ *is SPR,*

T2) The adaptation gains F_I *and* $F_P(t)$ *and the observation vector* $\Phi(t)$ *satisfy*

$$\sum_{t=0}^{t_1} \left[\Phi^T(t-1)\left(\frac{1}{2}F_I + F_P(t-1)\right)\Phi(t-1) - K\right]\nu^2(t) \ge 0 \tag{26}$$

for all $t_1 \geq 0$ *or*

$$\Phi^T(t)\left(\frac{1}{2}F_I + F_P(t)\right)\Phi(t) > K > 0, \qquad (27)$$

for all $t \geq 0$.

The proof is similar to that of Theorem 3.3 (p. 109) in [8] where Lemma 3.3 (p.110) is replaced by Lemma 3.1 of this paper.

It is interesting to note that condition (26) implies that the regressor vector has the property

$$\sum_{t=0}^{t_1}\left[\Phi^T(t-1)\Phi(t-1)\right] > \varepsilon > 0, \qquad (28)$$

which means that the trace of the covariance matrix of the regressor vector is positive, *i.e.*, that the energy of the signal is greater than zero (a milder condition than "persistence of excitation"). The magnitude of the proportional gain will depend on how far the transfer function is from a SPR transfer function (level of K) and what is the energy of the regressor (which depends upon the disturbance).

4. Experimental Results

The same AVC system as in [3] has been used to carry on the experiments. Description of the system and the characteristics of the various identified paths can be found in [3].

4.1. Broadband Disturbance Rejection Using Scalar Adaptation Gain

The adaptive feedforward filter structure for all of the experiments has been $n_R = 3$, $n_S = 4$ (total of 8 parameters). This complexity does not allow to verify the "perfect matching condition" (not enough parameters). A PRBS excitation on the global primary path will be considered as the disturbance.

For the *adaptive* operation the Algorithm *II* has been used with scalar adaptation gain ($\lambda_1(t) = 1$, $\lambda_2(t) = 0$)[3]. The experiments have been carried out by first applying the disturbance in open loop during $50s$ and after that closing the loop with the adaptive feedforward algorithms.

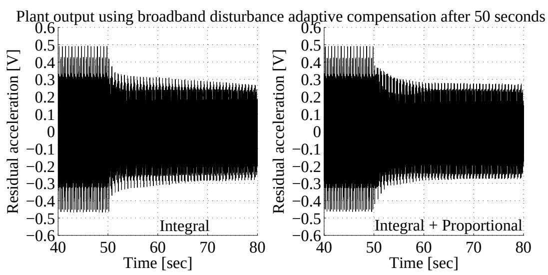

Figure 3: Real time results obtained with Algorithm II using "Integral" scalar adaptation gain (left) and "Integral + Proportional" scalar adaptation gain (right).

[3]Note that Algorithm *II* uses the same filtering as the FuLMS algorithm.

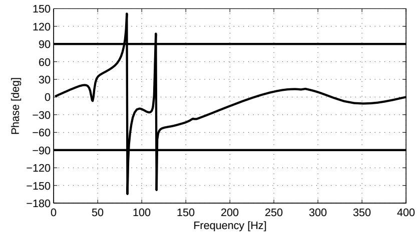

Figure 4: Phase of estimated $H(z^{-1})$ for Algorithm II.

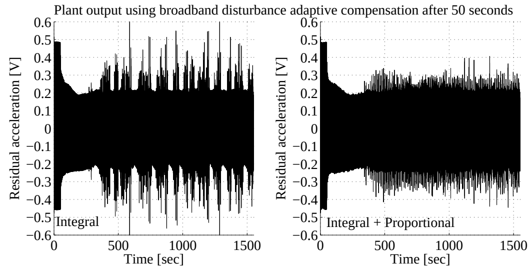

Figure 5: Real time results obtained with Algorithm II using "Integral" scalar adaptation gain (left) and "Integral + Proportional" scalar adaptation gain (right) over 1500s.

Time domain results obtained on the AVC system are shown in Fig. 3. The advantage of using an IP-PAA is an overall improvement of the transient behavior despite that the SPR condition on $H(q^{-1}) = \frac{A_M G}{P\hat{G}}$ is not satisfied (the SPR condition is not satisfied around 83 Hz and around 116 Hz as shown in Fig. 4). The improvement of performance can be explained by the relaxation of the SPR condition when using I+P adaptation. A variable $\alpha(t)$ in the IP-PAA has been chosen, starting with an initial value of 200 and linearly decreasing to 100 (over a horizon of 25s).

Fig. 5 shows the comparison between "Integral" and "Integral + Proportional" adaptation over an horizon of $1500s$ (Fig. 3 is a zoom of Fig. 5 covering only the first 30s after the introduction of the adaptive feedforward compensator). It is clear that IP adaptation gives better results even on a long run.

5. Conclusions

The paper has shown that the "Integral + Proportional" adaptation algorithms presented are useful in the context of adaptive feedforward vibration compensation. Theoretical development shows that the SPR condition can be relaxed and an improvement of the adaptation transients is obtained.

Appendix A. Proof of Result (15) - Lemma 2.1

Proof. This result can be directly obtained by applying Theorem 1 of [10]. Consider the equivalent feedback system associated with the I+P adaptation algorithm of eqs. (10) and eq. (13) shown in Fig. A.6. The same feedback system is represented in an alternate form in Fig. A.7, where the scalar gains $\frac{\lambda_2(t)}{2}$ and $\frac{\lambda_2}{2}$ are introduced to better understand the two classes of systems, $L(\lambda_2)$ and $N(\gamma)$, that will be used next (see also [10]). The class

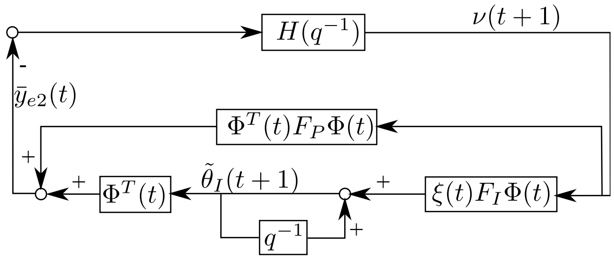

Figure A.6: Equivalent feedback representation of the PAA with "Integral + Proportional" adaptation.

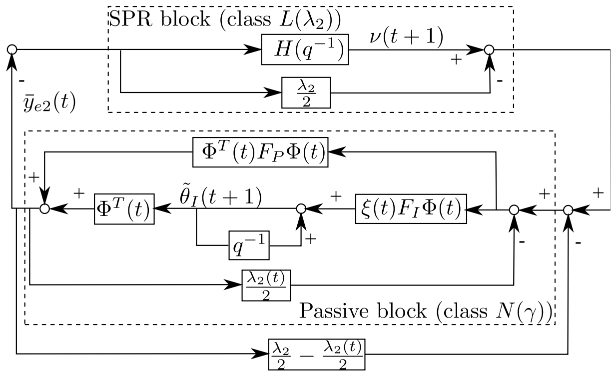

Figure A.7: Equivalent feedback representation of the PAA with "Integral + Proportional" adaptation and the representation of the $L(\lambda_2)$ and $N(\gamma)$ classes.

$L(\lambda_2)$ represents those LTI systems that in a parallel connection with a gain $-\frac{\lambda_2}{2}$ are strictly positive real (note that, from eq. (19), $\max_t \lambda_2(t) \leq \lambda_2$). On the other hand, the class $N(\gamma)$ denotes linear time-varying systems that in feedback connection with a block $\frac{\gamma(t)}{2}$ are passive (in this article $\gamma(t) = \lambda_2(t)$). Then considering the composite feedback block in Fig. A.7, it results that it is passive provided that $\lambda_2 \geq \lambda_2(t)$. Therefore, stability of the loop is assured since the linear equivalent feedforward path is SPR (see eq. (19)).

It remains to show that the feedback block indeed belongs to the class $N(\gamma)$, for $\gamma(t) = \lambda_2(t)$. One can verify this using Lemma 2 of [10] for the equivalent feedback path given by:

$$\tilde{\theta}_I(t) = \hat{\theta}_I(t) - \theta, \quad \text{(A.1)}$$
$$\nu(t+1) = -H(q^{-1})\Phi^T(t)\tilde{\theta}(t+1) \quad \text{(A.2)}$$
$$\tilde{\theta}_I(t+1) = \tilde{\theta}_I(t) + \xi(t)F_I(t)\Phi(t)\nu(t+1), \quad \text{(A.3)}$$
$$\bar{y}_{e2}(t) = \Phi^T(t)\tilde{\theta}_I(t) + \Phi^T(t)F(t)\Phi(t)\nu(t+1), \quad \text{(A.4)}$$

In order to use Lemma 2 of [10], one has to consider the following change of notation from (A.3) and (A.4)[4]:

$$A(t) = I,\ B(t) = \xi(t)F_I(t)\Phi(t),\ C(t) = \Phi^T(t), \text{ and} \quad \text{(A.5)}$$
$$D(t) = \Phi^T(t)F(t)\Phi(t). \quad \text{(A.6)}$$

[4] In [10], $k \equiv t$.

Then, eqs. (2.16)-(2.18) of Lemma 2 of [10] are satisfied for:

$$P(t) = F_I^{-1}(t),\ Q(t) = [1-\lambda_1(t)]F_I^{-1}(t), \quad \text{(A.7)}$$
$$S(t) = [1-\lambda_1(t)]\Phi(t), \quad \text{(A.8)}$$
$$R(t) = [2-\lambda_1(t)]f_{F_I}(t) + \frac{\lambda_2^2(t)}{\lambda_1(t)}f_{F_I}(t)f_{F_P}^2(t) + \lambda_2(t)f_{F_P}^2(t)$$
$$+ 2\frac{\lambda_2(t)}{\lambda_1(t)}f_{F_I}(t)f_{F_P}(t) + 2f_{F_P}(t), \quad \text{(A.9)}$$
$$f_{F_I}(t) \stackrel{\text{def}}{=} \Phi^T(t)F_I(t)\Phi(t),\ f_{F_P}(t) \stackrel{\text{def}}{=} \Phi^T(t)F_P(t)\Phi(t). \quad \text{(A.10)}$$

Finally, condition (2.21) of Theorem 1 of [10] is satisfied by the choice $\gamma(t) = \lambda_2(t)$ and the fact that the feedforward path is of the class $L(\lambda_2)$, where $\lambda_2 \geq \lambda_2(t)$ from eq. (19).

Thus the conditions of Theorem 1 from [10] are satisfied and the time-varying feedback system is asymptotically stable which implies eq. (15). □

Appendix B. Proof of Lemma 3.1

Proof. To analyse the strict positive realness of this transfer function, one has to check first that it's real part is strictly positive. We then have:

$$Re\{\frac{H(z^{-1})}{1+K\cdot H(z^{-1})}\} = \frac{K\cdot Re\{H\}^2 + Re\{H\} + K\cdot Im\{H\}^2}{(1+K\cdot Re\{H\})^2 + (K\cdot Im\{H\})^2}. \quad \text{(B.1)}$$

In eq. (B.1), the denominator is always strictly positive. Thus, the strict positive realness is satisfied if K is chosen such that the numerator of eq. (B.1) is also strictly positive. This is always true if K satisfies the relation

$$K > -\frac{Re\{H(e^{-j\omega})\}}{Re\{H(e^{-j\omega})\}^2 + Im\{H(e^{-j\omega})\}^2},\ 0 \leq \omega \leq \pi \cdot f_S,$$

f_S being the sampling frequency.

Next, the stability of $H/(1+KH)$ is analyzed. Under hypothesis H2, the poles of $H/(1+KH)$ are given by the roots of the polynomial

$$P(q^{-1}) = 1 + \frac{\sum_{p=1}^{n_A} a_p q^{-p} + K\sum_{m=1}^{n_B} b_m q^{-m}}{1+Kb_0} \quad \text{(B.2)}$$

and assuming K large enough such that $Kb_m \gg a_p$, $\forall m \in \{1,\ldots,n_B\}$, $p \in \{1,\ldots,n_A\}$, $P(q^{-1}) \cong$

$$\cong \begin{cases} 1 + \sum_{m=1}^{n_B} \frac{b_m}{b_0} q^{-m} & \text{if } n_B \geq n_A, \\ 1 + \sum_{m=1}^{n_B} \frac{b_m}{b_0} q^{-m} + \sum_{p=n_B+1}^{n_A} \frac{a_p}{1+Kb_0} q^{-p} & \text{if } n_B < n_A. \end{cases}$$

Thus for $n_B \geq n_A$, the poles and the zeros of $H/(1+KH)$ become identical when $K \to \infty$. For $n_B < n_A$, in addition to the poles identical to the zeros of $B(q^{-1})$, $n_A - n_B$ poles appear that go to zero as $K \to \infty$. The hypothesis H1 has been introduced to assure the stability of the direct path when H2 is satisfied. Hypothesis H2 is necessary since if $b_0 = 0$, $H/(1+KH)$ becomes unstable for large K.

References

[1] Jacobson, C., Johnson, C., Cormick, D.M., Sethares, W.. Stability of active noise control algorithms. IEEE Signal Processing letters 2001;8(3):74–76.

[2] Wang, A., Ren, W.. Convergence analysis of the filtered-u algorithm for active noise control. Signal Processing 1999;73:255–266.

[3] Landau, I., Alma, M., Airimiţoaie, T.. Adaptive feedforward compensation algorithms for active vibration control with mechanical coupling. Automatica 2011;47(10):2185 – 2196. doi: 10.1016/j.automatica.2011.08.015.

[4] Landau, I., Airimiţoaie, T., Alma, M.. An iir youla-kucera parametrized adaptive feedforward compensator for active vibration control with mechanical coupling. In: Proceedings of the 50th IEEE Conference on Decision and Control, Atlanta, USA. 2011, p. 6777–6782.

[5] Montazeri, A., Poshtan, J.. A computationally efficient adaptive iir solution to active noise and vibration control systems. IEEE Trans on Automatic Control 2010;AC-55:2671 – 2676.

[6] Anderson, B., Bitmead, R., Johnson, C., Kokotovic, P., Kosut, R., Mareels, I., et al. Stability of adaptive systems. Cambridge Massachusetts , London, England: The M.I.T Press; 1986.

[7] Tomizuka, M.. Parallel mras without compensation block. Automatic Control, IEEE Transactions on 1982;27(2):505 – 506. doi: 10.1109/TAC.1982.1102907.

[8] Landau, I.D., Lozano, R., M'Saad, M., Karimi, A.. Adaptive control. London: Springer; 2nd ed.; 2011.

[9] Landau, I.D.. Adaptive control, The model reference approach. New York and Basel: Marcel Dekker, Inc.; 1st ed.; 1979.

[10] Landau, I., Silveira, H.. A stability theorem with applications to adaptive control. Automatic Control, IEEE Transactions on 1979;24(2):305 – 312. doi:10.1109/TAC.1979.1102009.

[11] Landau, I.. An extension of a stability theorem applicable to adaptive control. Automatic Control, IEEE Transactions on 1980;25(4):814 – 817. doi:10.1109/TAC.1980.1102440.